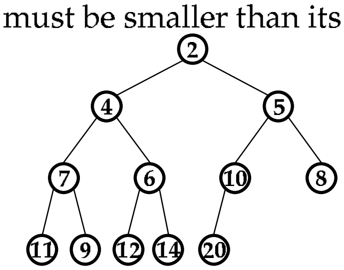
CSE 373: Heaps
(other operations and variations)

Chapter 6

Heaps: Quick Recap

Heaps:

- structure is a complete binary tree
- each node must be smaller than its descendants

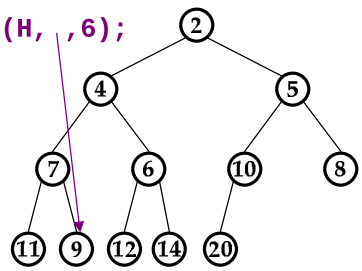


- main operations are **`Insert()`** and **`DeleteMin()`**
- heaps have an compact array-based representation

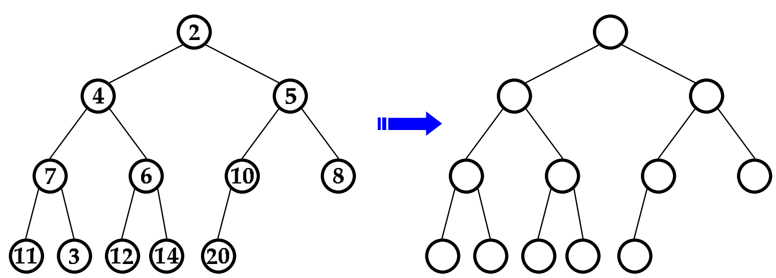
Other Operations: `DecreaseKey()`

`DecreaseKey()` – lowers a node's value
(while preserving heap ordering)

`DecreaseKey(H, ,6);`



`DecreaseKey()` – Continued

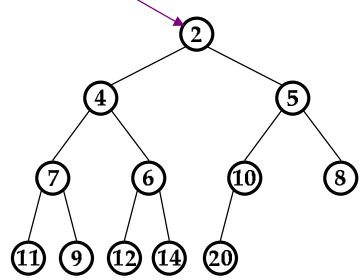


running time?

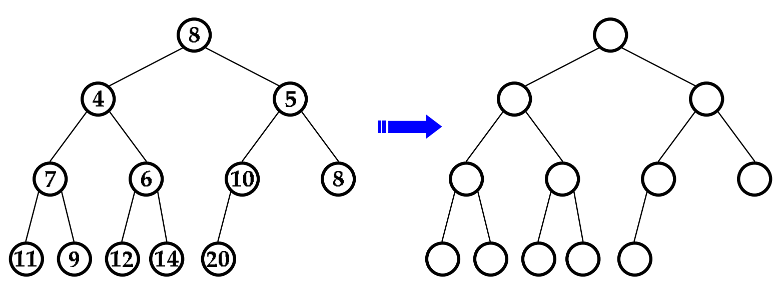
`IncreaseKey()`

`IncreaseKey()` – raises a node's value

```
IncreaseKey(H, ,6);
```



`IncreaseKey()` – Continued

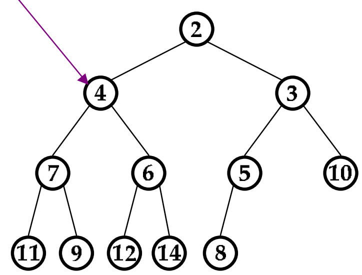


running time?

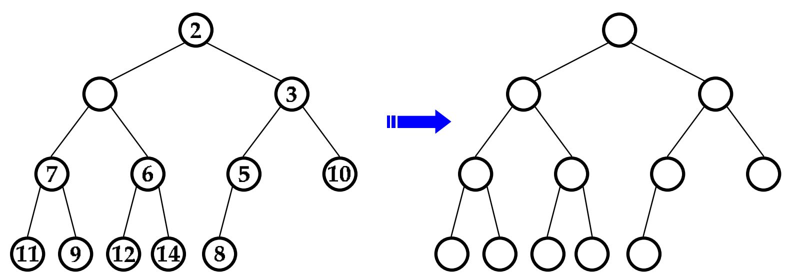
Delete()

Delete() – removes a node from the heap

`Delete(H, ,6);`



Delete() – Continued



running time?

Let’s Write a Heap Routine…

BuildHeap()

BuildHeap() – creates a heap from an array

12	5	11	3	10	6	9	4	8	1	7	2

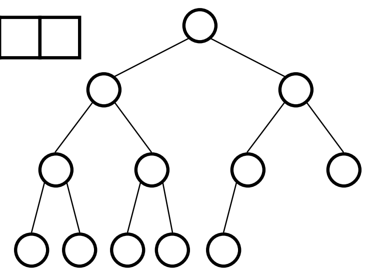
Straightforward Implementation: **Insert()** elements into an empty heap one at a time

running time?

`BuildHeap()` – Continued

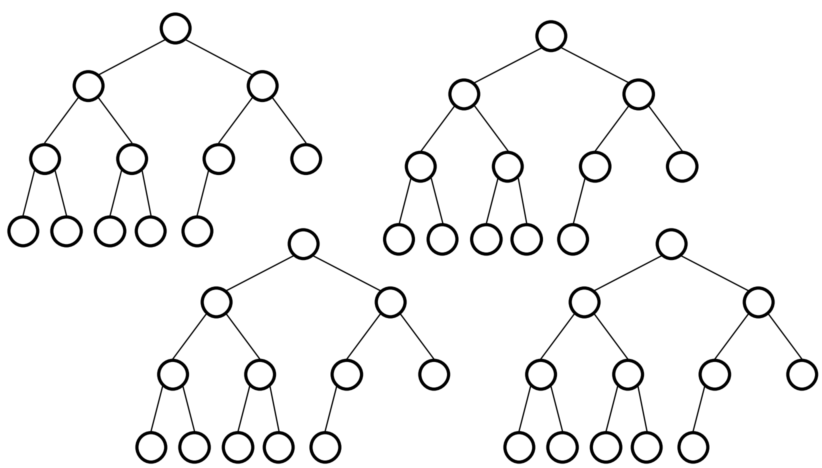
Better Implementation: Treat input array as a heap and “percolate down” first $n/2$ values

12	5	11	3	10	6	9	4	8	1	7	2

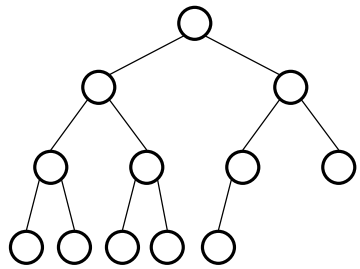


running time?

`BuildHeap()` – even more



`BuildHeap()` running time



MaxHeaps

MaxHeaps: the dual of the Heaps we've defined
- support fast `Insert()` and `DeleteMax()` ops
- work exactly the same as (Min)Heaps

Why is `DeleteMax()` expensive on a normal heap?

What's the running time?

d-Heaps

d-Heaps: Just like normal heaps but with *d* children rather than 2

Intuition: tree will be shallower so ops will be faster

However…

- more comparisons need to be made when percolating down
- if *d* not a power of 2, finding parent/children will be slower

What about asymptotic running time?

Bottom Line: 4-heaps ***may*** outperform 2-heaps

Merging Heaps

How to merge heaps effectively?

Straightforward method: copy both arrays into a single array and use **`BuildHeap()`**

running time?

Advanced methods:

- pointer-based imbalanced heaps
  - *leftist heaps* – a bit like AVL trees; O(log *n*) merge
  - *skew heaps* – like Splay trees; O(log *n*) amortized ops
  - *binomial queues* – O(log *n*) merge, but ~O(1) insert