CSE 373: Sorting

Chapter 7

Introduction to Sorting

Sorting: One of the most fundamental algorithms

Input: An array A[] of values and its size, n.

Output: The array stored in sorted order:

if $i < j$ then $A[i] \le A[j]$, $\forall i,j \le n$

Goals: sort as quickly as possible

ideally, use O(1) memory (other than A[])

handle pre-sorted lists quickly

Insertion Sort

Insertion Sort: One of the simplest sorting algorithms, based on List ADT `Insert()`.

- n-1 passes
- after pass i, elements 0..i will be in sorted order
- in pass i, we ripple the i^{th} element down the array until it's sorted (with respect to elements 0..i-1)

Insertion Sort Example

position:	*0*	*1*	*2*	*3*	*4*	*5*
input:	**7**	**4**	**9**	**5**	**8**	**2**
pass 1:						
pass 2:						
pass 3:						
pass 4:						
pass 5:						

Insertion Sort Analysis

- Why ripple down rather than up?
- Best case input? Running time?
- Worst case input? Running time?

Adjacent Swap Algorithms

A class of algorithms that sort simply by comparing and swapping adjacent elements

- Insertion Sort
- Bubble Sort
- Selection Sort

Inversions

- Given A[], an *inversion* is a pair (i,j) such that $i < j$, but $A[i] > A[j]$.
  - How many inversions in our example?

    7 4 9 5 8 2

- The number of inversions in A[] equals the number of adjacent swaps required to sort it
  - Why?

Average Case Analysis

Q: What is the average number of inversions in a random input array?

A: Consider an arbitrary list L with n unique values

Consider the reversal of the list L_R

Every pair (i,j) represents an inversion in L or in L_R

The total number of distinct (i,j) pairs is $n(n-1)/2$

On average, half of these will be in L, half will be in L_R

Thus, the average array has $n(n-1)/4$ inversions

So, adjacent swap algorithms run in $\Theta(n^2)$ on average

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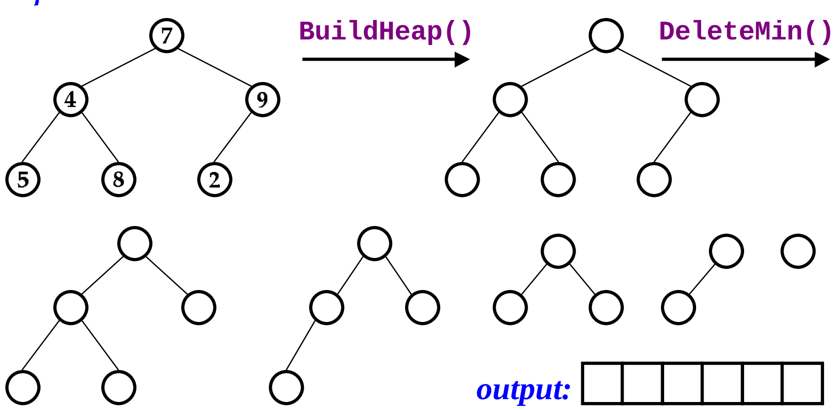
Heapsort

- Naive algorithm:
  - Run **`BuildHeap()`** on the input array
  - Call **`DeleteMin()`** n times, storing the results in an output array
- Running Time?
- Disadvantage?
- How can we fix this?

Heapsort Example

input: **7 4 9 5 8 2**



Improved Heapsort

- Use the heap's array to store the sorted values
- *Recall:* a *k*-element heap uses the first *k* positions of its implementing array
- Thus, whenever we delete an element from the heap, store it at the end of the array
- What does this give us?
- How to fix it?

Treesort?

- BSTs can obviously be used to sort input
  - `Insert()` all values
  - traverse tree in-order, copying to output array
- This is rarely done in practice (unless a tree is already being used to store the data)
  - asymptotically similar to Heapsort
  - *but* trees require more memory
  - *and* can't be done using only input array memory
  - might as well use Heapsort