# CSE 373: Sorting

*(Shellsort and Mergesort)*

Chapter 7

## Drawback of Adjacent Swap Sorts

- Each swap only fixes a single inversion
- Thus, elements that are far out of place must be swapped with many values instead of being moved into place more directly:

  **4 5 7 8 9 2**

- This is the motivation for Shellsort (*named after its inventor, Donald Shell*): try to move values to their general area quickly, then fix them up

# Shellsort

- Uses $p$ phases
- The phases are characterized by an *increment sequence* of integers: $h_1, h_2, h_3, \ldots, h_p$:
  - Typically, $h_i > h_{i+1}$
  - $h_p = 1$ (last phase is insertion sort)
- In phase $k$, we compare and swap values that are $h_k$ positions apart until they are sorted
- This essentially performs $h_k$ independent insertion sorts in phase $k$



# Shellsort Example

*h* = 5, 3, 2, 1 *input* = 7, 4, 8, 6, 9, 2, 5, 3

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 4 | 8 | 6 | 9 | 2 | 5 | 3 |
| 2 | 4 | 3 | 6 | 9 | 7 | 5 | 8 |
| 2 | 4 | 3 | 6 | 9 | 7 | 5 | 8 |
| 2 | 4 | 3 | 5 | 8 | 7 | 6 | 9 |
| 2 | 4 | 3 | 5 | 8 | 7 | 6 | 9 |
| 2 | 4 | 3 | 5 | 6 | 7 | 8 | 9 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

## Increment Sequences

- Designing increment sequences:
  - Running time is proportional to the number of increments, so we don't want too many
  - But just having one would give us insertion sort
- Worst-case running time:

  $\Sigma_i (h_i (n/h_i)^2)$: $h_i$ insertion sorts of $n/h_i$ elements each;

  (recall: insertion sort has worst-case of $O(n^2)$)



## Common Increment Sequences

- Shell's original sequence:

  $h = n/2, n/4, n/8, \ldots, 2, 1$
  - probably the most intuitive sequence
  - but, it has a worst-case of $O(n^2)$
- Hibbard's sequence:

  $h = 2^k$-1, …, 15, 7, 3, 1
  - adjacent numbers are relatively prime
  - leads to a worst-case of $O(n^{1.5})$

# The Merge Operation

- Given two sorted lists, **`Merge()`** combines them into a single sorted list:

| | | | |
|---|---|---|---|
| 4 | 6 | 7 | 8 |

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 9 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

- Running time of **`Merge()`**?



# Mergesort

Elegant recursive sorting algorithm:

- if the input is one element, it's sorted; return
- otherwise, split the input into two equal-sized lists
- call **`Mergesort()`** recursively on each list
- merge the sorted lists that are returned

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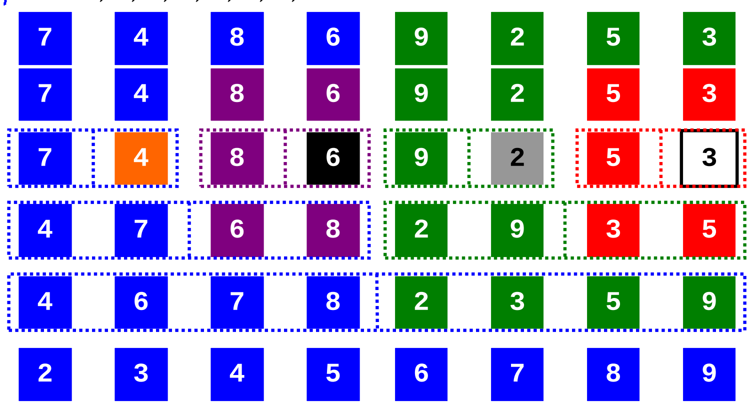
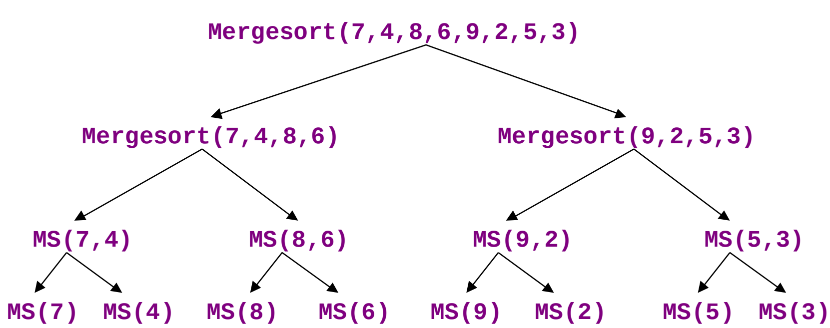
# Mergesort Example

*input* = 7, 4, 8, 6, 9, 2, 5, 3

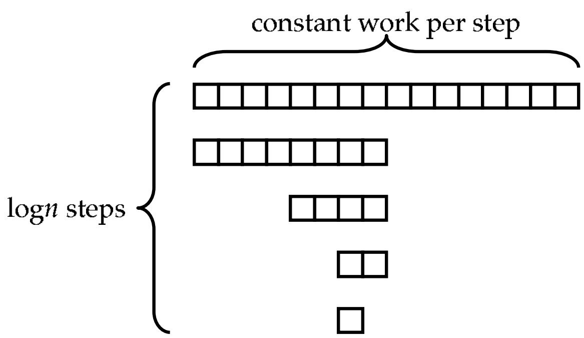





# Mergesort Call Tree



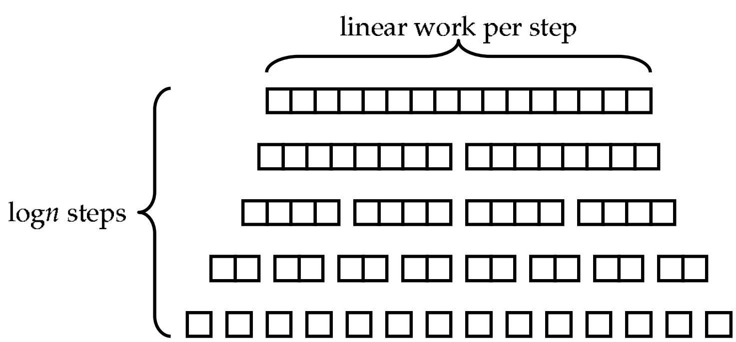
# Binary Search Running Time




$= O(1) \times O(\log n) = O(\log n)$



# Mergesort Running Time




$= O(n) \times O(\log n) = O(n \log n)$

Disadvantages?

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