Static Single-Assignment Form

- Overview of SSA IR
  - Constructing SSA graphs
  - SSA-based optimizations
  - Converting back from SSA form

- Source: Appel ch. 19, also an extended discussion in Cooper-Torczon sec. 9.3

Def-Use (DU) Chains

- Common dataflow analysis problem: Find all sites where a variable is used, or find the definition site of a variable used in an expression
- Traditional solution: def-use chains – additional data structure on top of the dataflow graph
  - Link each statement defining a variable to all statements that use it
  - Link each use of a variable to its definition

DU-Chain Drawbacks

- Expensive: if a typical variable has N uses and M definitions, the total cost is O(N * M)
  - Would be nice if cost were proportional to the size of the program
- Unrelated uses of the same variable are mixed together
  - Complicates analysis

SSA: Static Single Assignment

- IR where each variable has only one definition in the program text
  - This is a single *static* definition, but that definition can be in a loop that is executed dynamically many times

SSA within Basic Blocks

Similar to the *local value numbering* optimization

- Original

  a := b + c

  b := a – d

  c := b + c

  d := a – d

- SSA

  $a_1 := b_0 + c_0$

  $b_1 := a_1 - d_0$

  $c_1 := b_1 + c_0$

  $d_1 := a_1 - d_0$

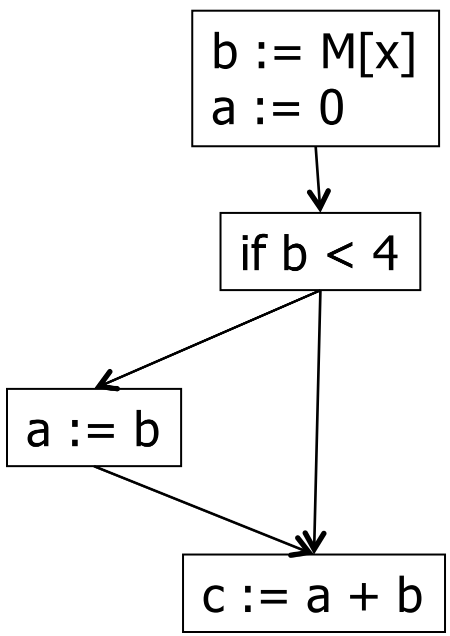
Merge Points

- The issue is how to handle merge points
- Solution: introduce a Φ-function

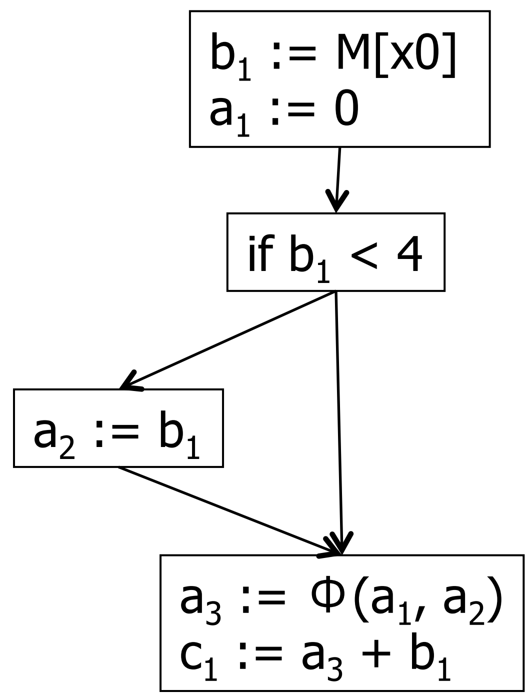
  $a_3 := \Phi(a_1, a_2)$
- Meaning: a_3 is assigned either a_1 or a_2 depending on which control path is used to reach the Φ-function

Example

Original



SSA

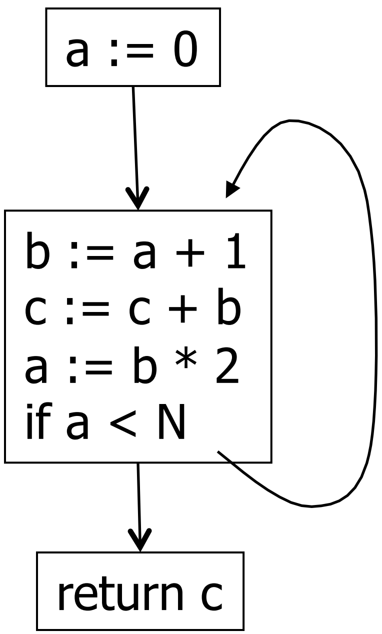


How Does Φ "Know" What to Pick?

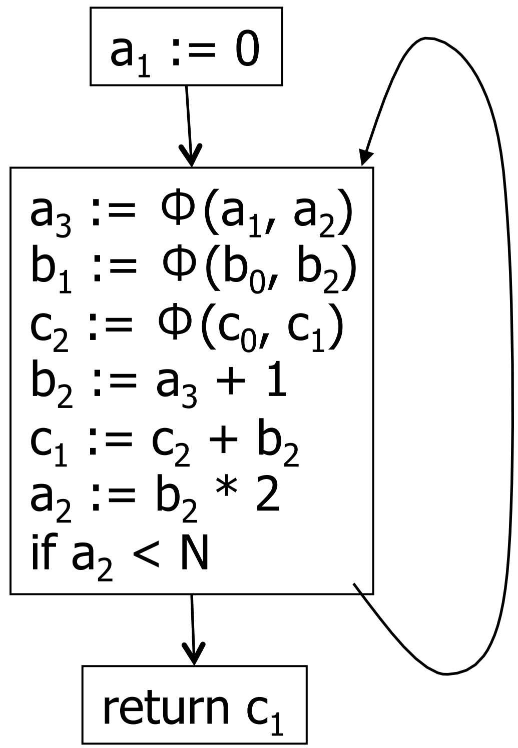
- It doesn't
  - When we translate the program to executable form, we can add code to copy either value to a common location on each incoming edge
  - For analysis, all we may need to know is the connection of uses to definitions – no need to "execute" anything

Example With Loop

Original



SSA



Notes:

- a_0, b_0, c_0 are initial values of a, b, c on block entry
- b_1 is dead – can delete later
- c is live on entry – either input parameter or uninitialized

Converting To SSA Form

- Basic idea
  - First, add Φ-functions
  - Then, rename all definitions and uses of variables by adding subscripts
- Could simply add Φ-functions for every variable at every join point(!)
  - Wastes *way* too much space and time
  - Not needed

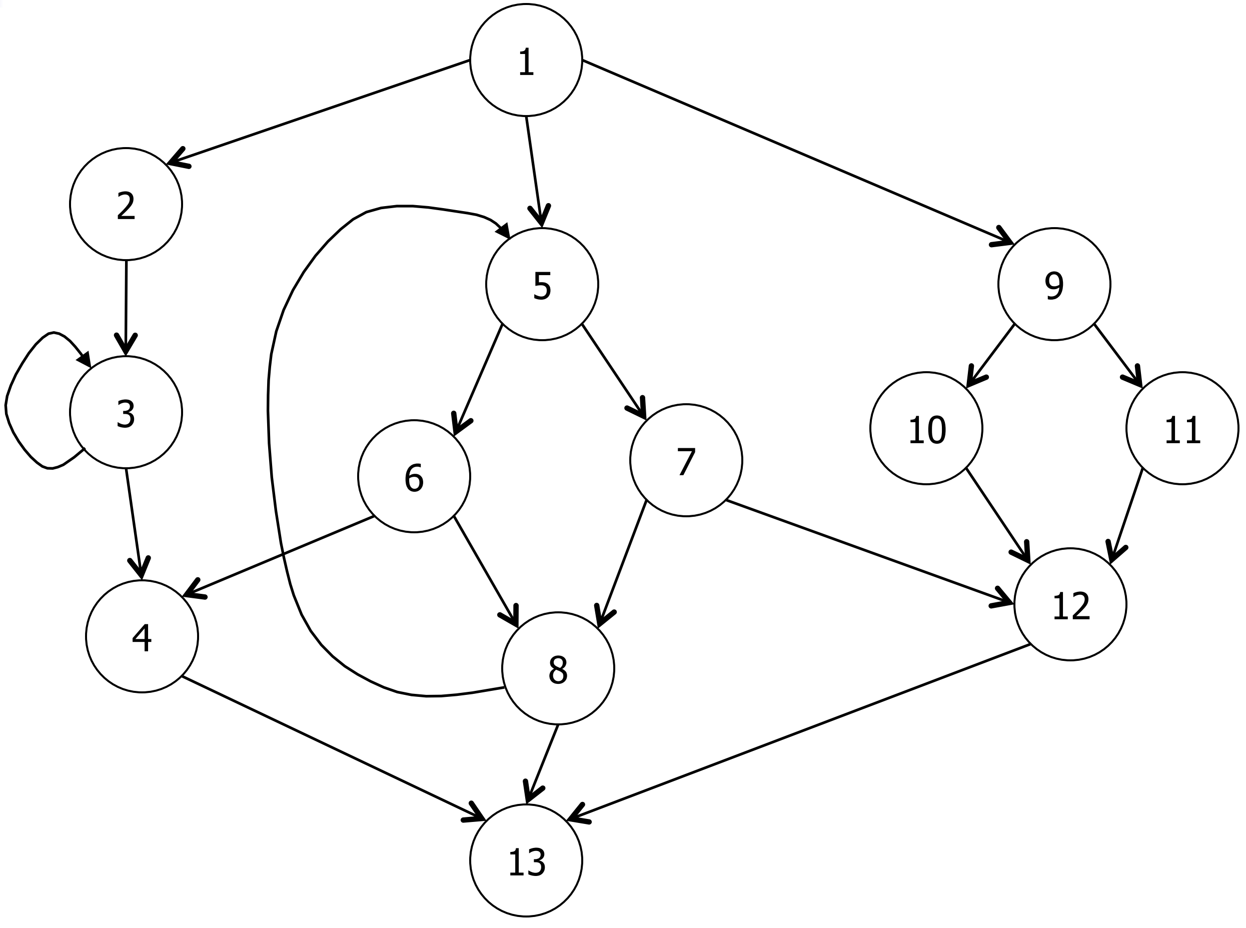
Path-convergence criterion

- Insert a Φ-function for variable a at point z when:
  - There are blocks x and y, both containing definitions of a, and $x \neq y$
  - There are nonempty paths from x to z and from y to z
  - These paths have no common nodes other than z
  - z is not in both paths prior to the end (it may appear in one of them)

Details

- The start node of the flow graph is considered to define every variable (even if to "undefined")
- Each Φ-function itself defines a variable, so we need to keep adding Φ-functions until things converge

Dominators



Dominators and SSA

- One property of SSA is that definitions dominate uses; more specifically:
  - If x := $\Phi(\ldots,x_i,\ldots)$ is in block n, then the definition of x_i dominates the i^{th} predecessor of n
  - If x is used in a non-Φ statement in block n, then the definition of x dominates block n

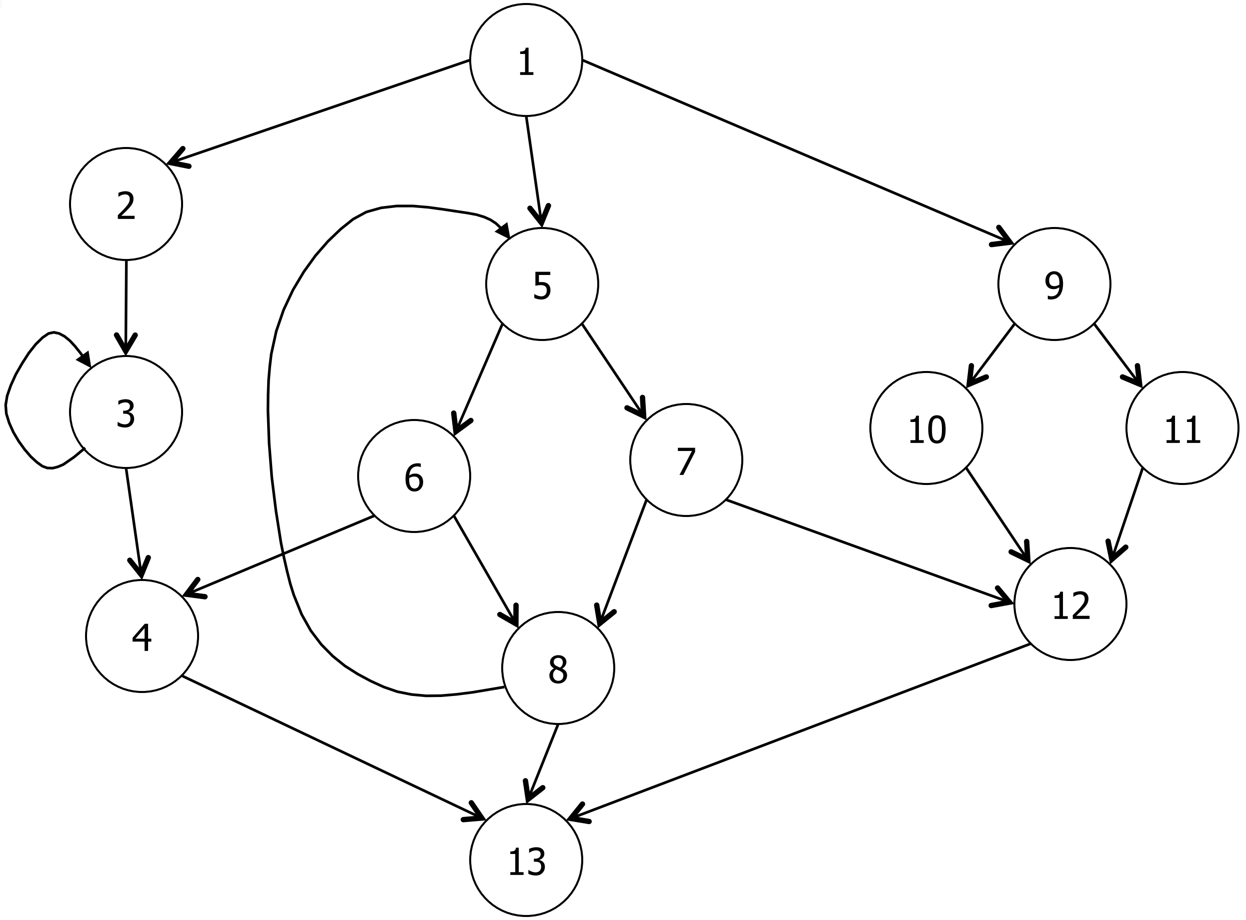
Dominance Frontiers

- To get a practical algorithm for placing Φ-functions, we need to avoid looking at all combinations of nodes leading from x to y
- Instead, use the dominator tree in the flow graph

Dominance Frontiers

- Definitions
  - x *strictly dominates* y if x dominates y and x ≠ y
  - The *dominance frontier* of a node x is the set of all nodes w such that x dominates a predecessor of w, but x does not strictly dominate w
- Essentially, the dominance frontier is the border between dominated and undominated nodes

Dominance Frontiers



Dominance Frontier Criterion

- If a node x contains the definition of variable a, then every node in the dominance frontier of x needs a Φ-function for a
  - Since the Φ-function itself is a definition, this needs to be iterated until it reaches a fixed-point
- Theorem: this algorithm places exactly the same set of Φ-functions as the path criterion given previously

Placing Φ-Functions: Details

- The basic steps are:
  1. Compute the dominance frontiers for each node in the flowgraph
  2. Insert just enough Φ-functions to satisfy the criterion. Use a worklist algorithm to avoid reexamining nodes unnecessarily
  3. Walk the dominator tree and rename the different definitions of variable a to be a_1, a_2, a_3, …

SSA Optimizations

- Given the SSA form, what can we do with it?
- First, what do we know? (i.e., what information is kept in the SSA graph?)

Dead-Code Elimination

- A variable is live iff its list of uses is not empty
- Algorithm to delete dead code:

  while there is some variable v with no uses

  if the statement that defines v has no other side effects, then delete it

  - Need to remove this statement from the list of uses for its operand variables – which may cause those variables to become dead

Simple Constant Propagation

- If c is a constant in v := c, any use of v can be replaced by c
  - Then update every use of v to use constant c
- If the c_is in v := $\Phi(c_1, c_2, \ldots, c_n)$ are all the same constant c, we can replace this with v := c
- Can also incorporate copy propagation, constant folding, and others in the same worklist algorithm

Simple Constant Propagation

```
W := list of all statements in SSA program
while W is not empty
   remove some statement S from W
   if S is v:=Φ(c, c, …, c), replace S with v:=c
   if S is v:=c
      delete S from the program
      for each statement T that uses v
         substitute c for v in T
         add T to W
```

Converting Back from SSA

- The meaning of $x := \Phi(x_1, x_2, \ldots, x_n)$ is "set $x := x_1$ if arriving on edge 1, set $x := x_2$ if arriving on edge 2, etc."
- So, for each i, insert $x := x_i$ at the end of predecessor block I
- Issues: lost copies (recall from class; C&T sec 9.3), swap (C&T sec 9.3)

SSA Summary

- Combine information from control flow graph and data flow analysis
- Allows efficient implementation of many optimizations (eg constant propagation, dead code elimination, common subexpression elimination)
- Some optimizations are more efficient without SSA (eg most loop transformations —fission, fusion, unfolding…)