# Single Static Assignment

CSE 401 Section 10/10

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Adapted from Laura Vonesson’s Wi17 Slides

# The Final Stretch

 You are here

| SUN | MON | TUE | WED | THU | FRI | SAT |
|---|---|---|---|---|---|---|
| | | | | <br>Compiler Additions | | Report<br><br>M501 Additions |
| M501 Report<br><br>Evals!! | | | Review Session<br>(4:30 EEB 045) | **Final Exam**<br>(8:30) | **Eternal Mastery of Compilers** | |

# Problem 1

(review of dataflow)

# Single Static Assignment

- An intermediate representation where each variable has only one definition:

| **Original** | **SSA Form** |
|---|---|
| `a := x + y` | $a_1 := x_1 + y_1$ |
| `b := a - 1` | $b_1 := a_1 - 1$ |
| `a := y + b` | $a_2 := y_1 + b_1$ |
| `b := x * 4` | $b_2 := x_1 * 4$ |
| `a := a + b` | $a_3 := a_2 + b_2$ |

# SSA: Why We Love It

- Without SSA, all definitions and uses of a variable get mixed together
    - Computing information about the definitions of a variable is an expensive but necessary part of many dataflow analyses

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- Without SSA, all definitions and uses of a variable get mixed together
    - Computing information about the definitions of a variable is an expensive but necessary part of many dataflow analyses

- Doing the work of converting to SSA once makes many analyses + optimizations more efficient
    - SSA can be thought of as an implicit representation of Definition/Use chains

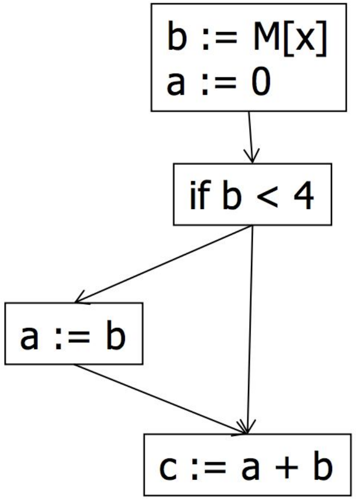
# SSA: Why We Love It

- Ex: Dead Store Elimination
  - Without SSA: Compute live variables at every point, which requires working backwards and using the dataflow sets to check for *any path* that does not kill the variable, and eliminate any stores that are not to a live variable.
  - With SSA: Eliminate any store where the variable being assigned has 0 uses.

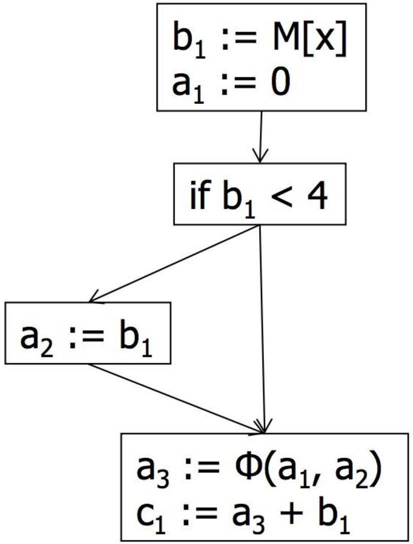
# Phi-Functions

- A method of representing an uncertain value for a certain definition
  - Not a “real” instruction -- only a formality needed for SSA

**Original**



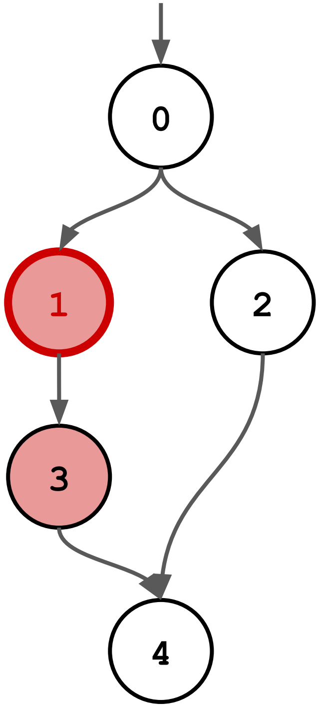

**SSA Form**



# Dominators

- A node **`X`** ***dominates*** a node **`Y`** iff every path from the entry point of the control flow graph to **`Y`** includes **`X`**

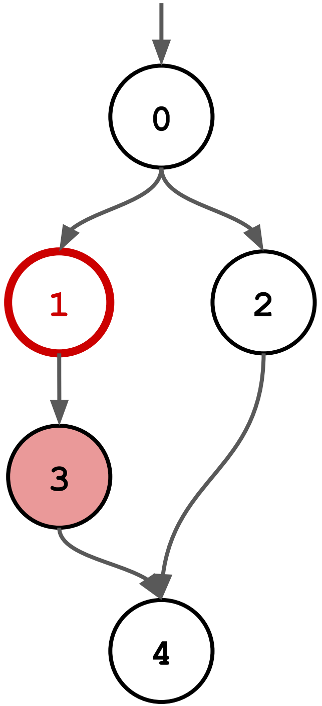
Node 1 dominates nodes 1 and 3.
It does not dominate 4 because there is another path that reaches it.



# Strict Dominance

- A node `X` ***strictly dominates*** a node `Y` if `X` dominates `Y` and `X` ≠ `Y`.

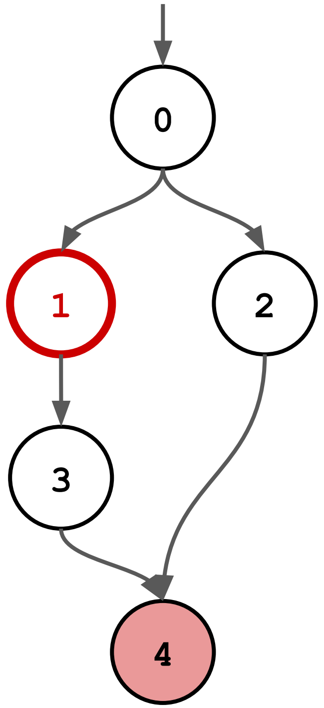
Node 1 only strictly dominates node 3 because it is the only dominated node that is not equal to 1.



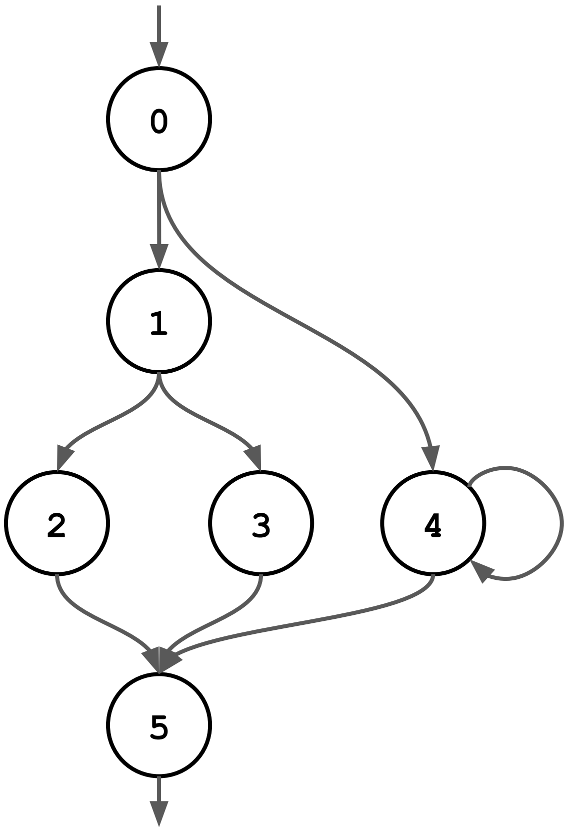
# Dominance Frontiers

- A node **`Y`** is in the ***dominance frontier*** of node **`X`** if **`X`** dominates an immediate predecessor of **`Y`** but **`X`** does not strictly dominate **`Y`**.
- Essentially, the border between dominated and non-dominated nodes
    - Note: a node can be in its own dominance frontier
- This is where phi function merging is necessary

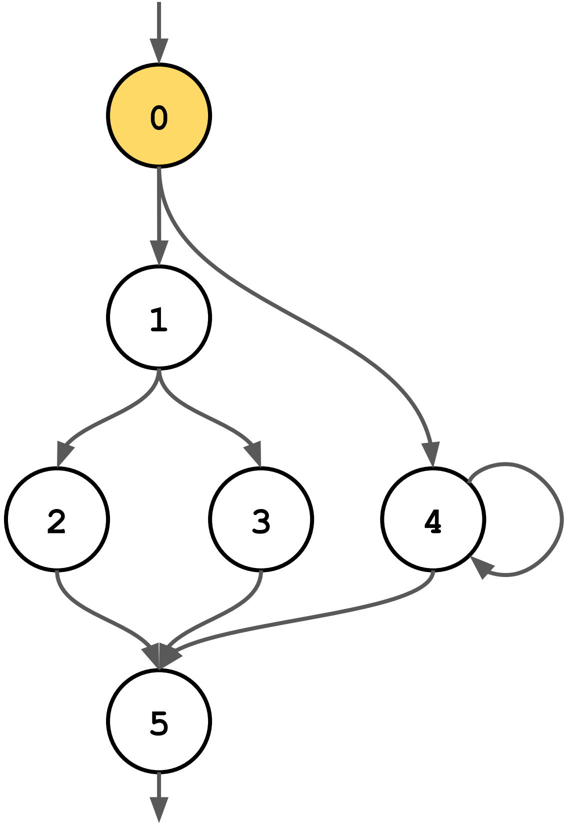
Node 4 is in the dominance frontier of node 1 because an immediate predecessor (node 3) is dominated by 1.



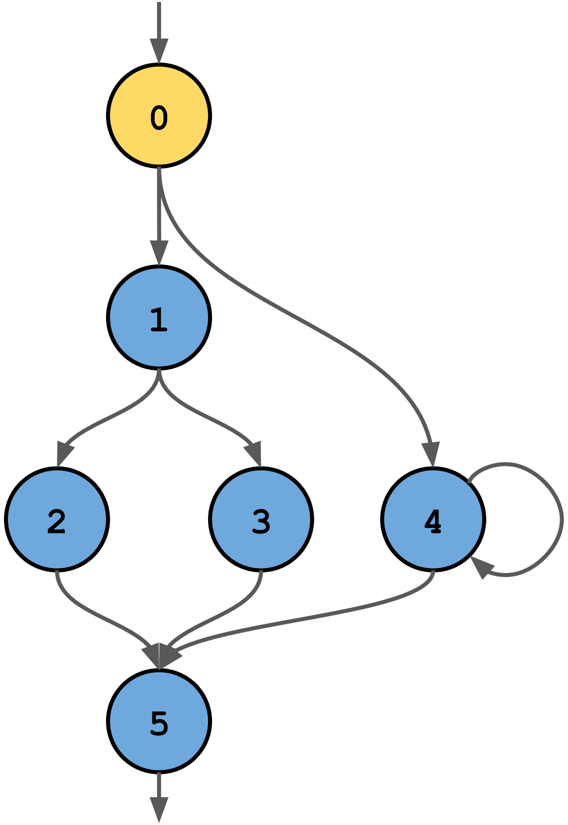
# Problem 2



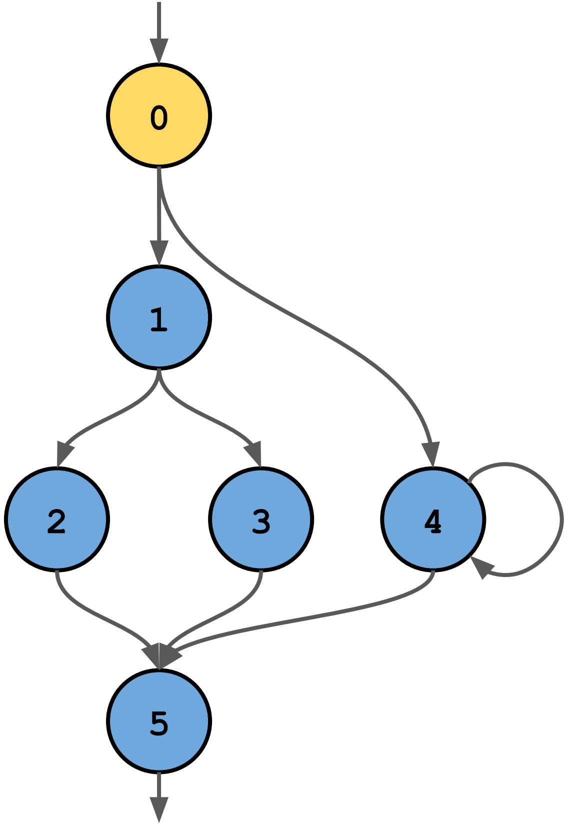

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



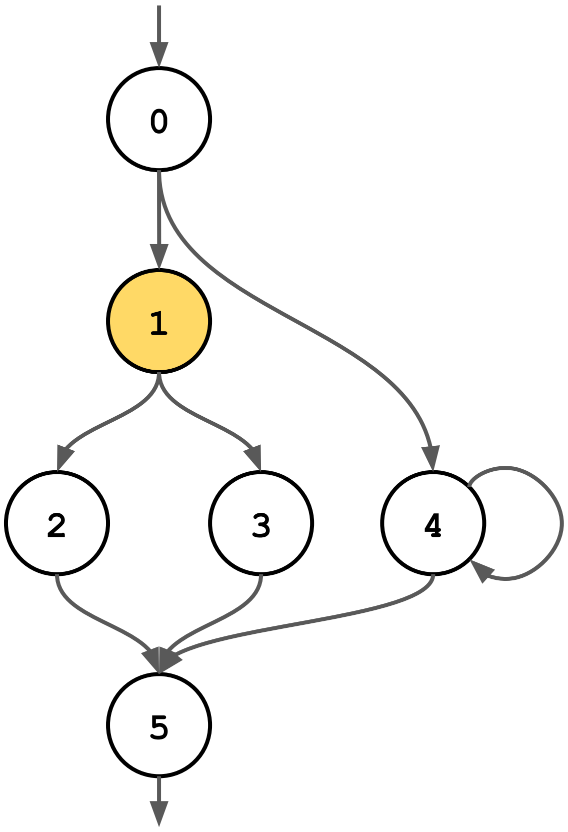

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



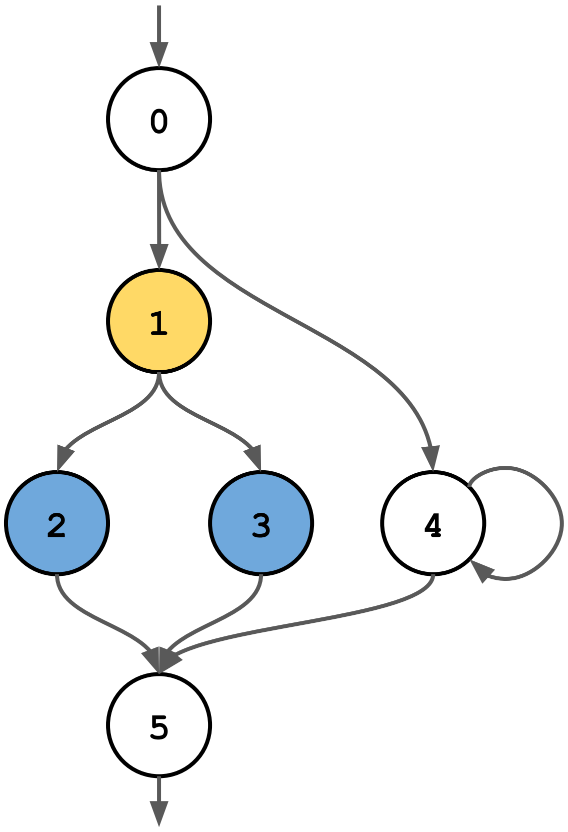

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



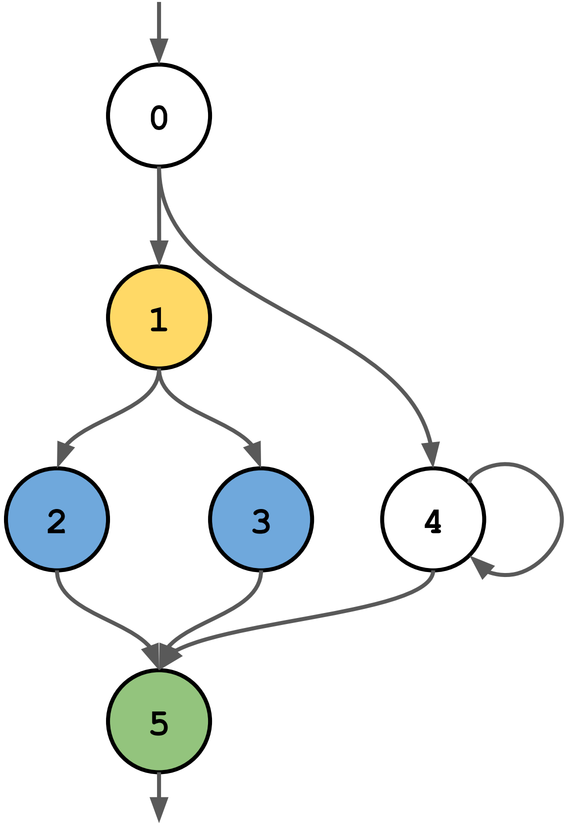

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



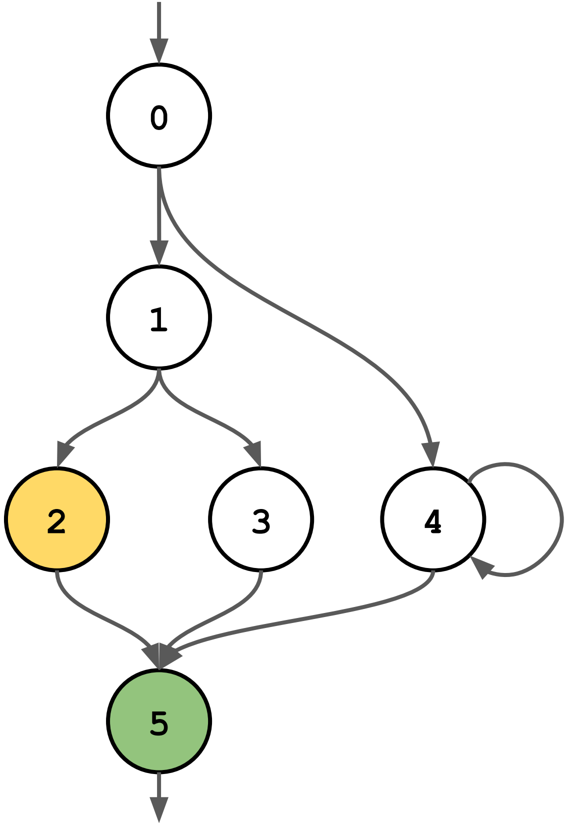

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



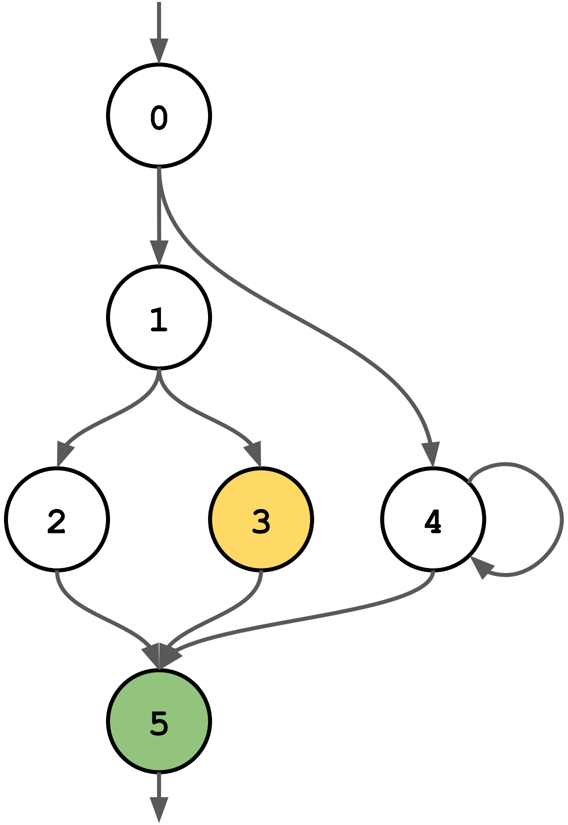

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



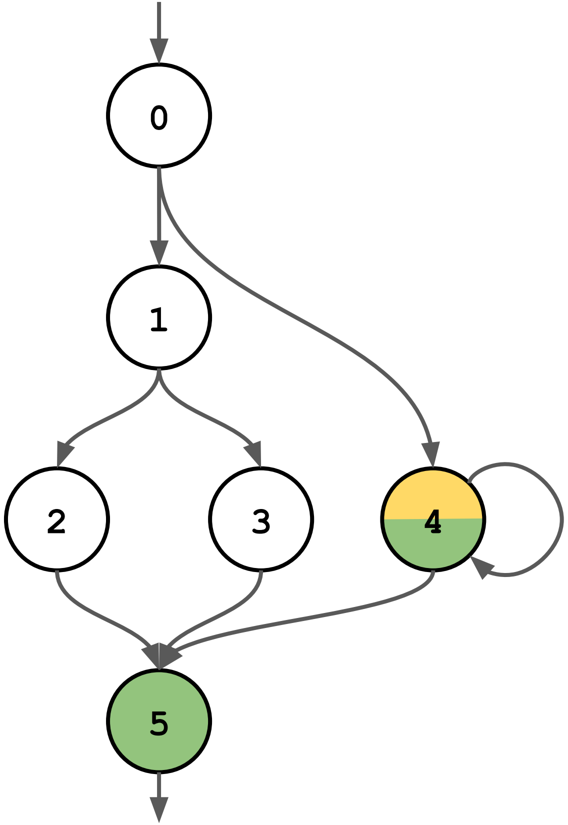

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | 5 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |



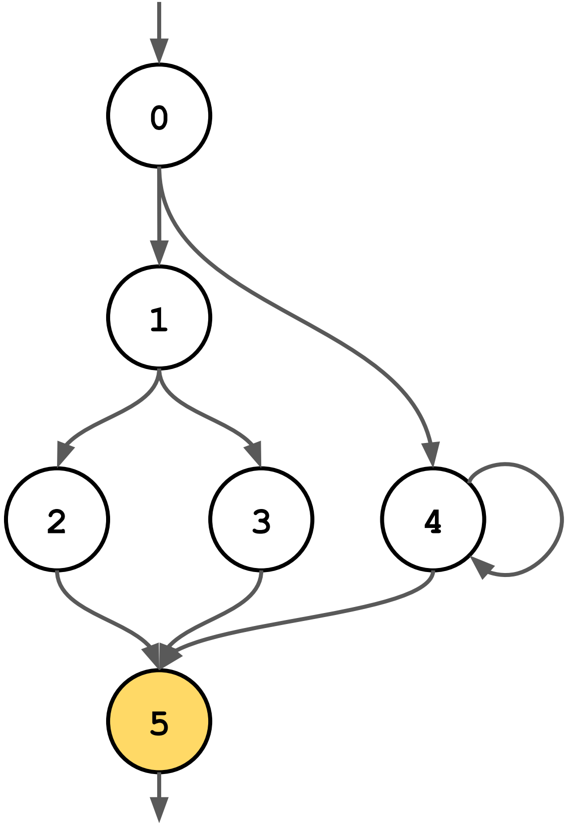

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | 5 |
| 2 | ∅ | 5 |
| 3 | | |
| 4 | | |
| 5 | | |




| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
| --- | --- | --- |
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | 5 |
| 2 | ∅ | 5 |
| 3 | ∅ | 5 |
| 4 | | |
| 5 | | |

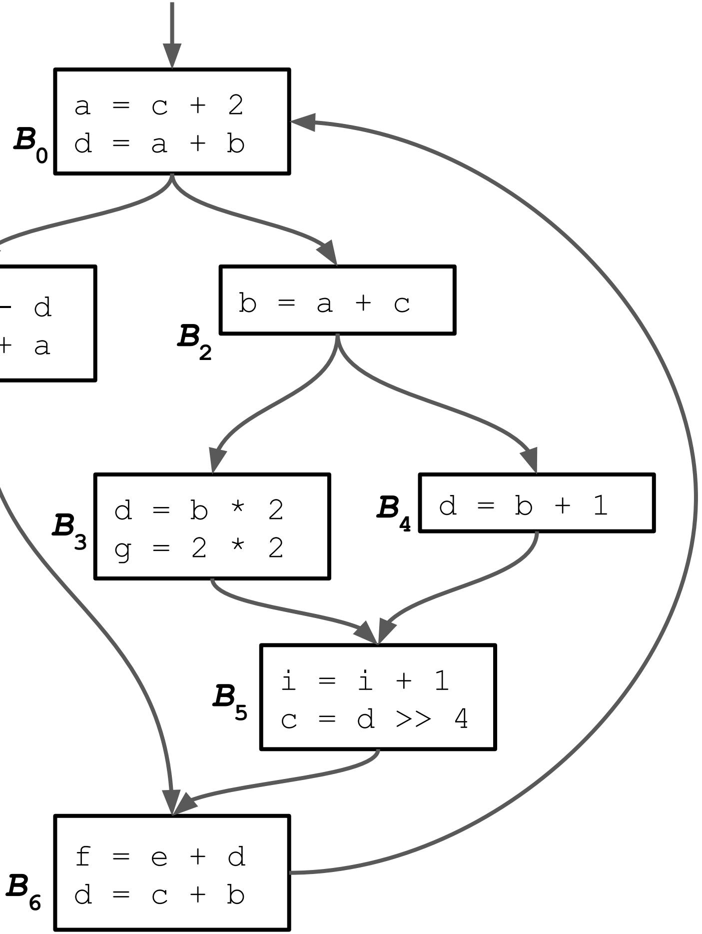



| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
| --- | --- | --- |
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | 5 |
| 2 | ∅ | 5 |
| 3 | ∅ | 5 |
| 4 | ∅ | 4, 5 |
| 5 | | |



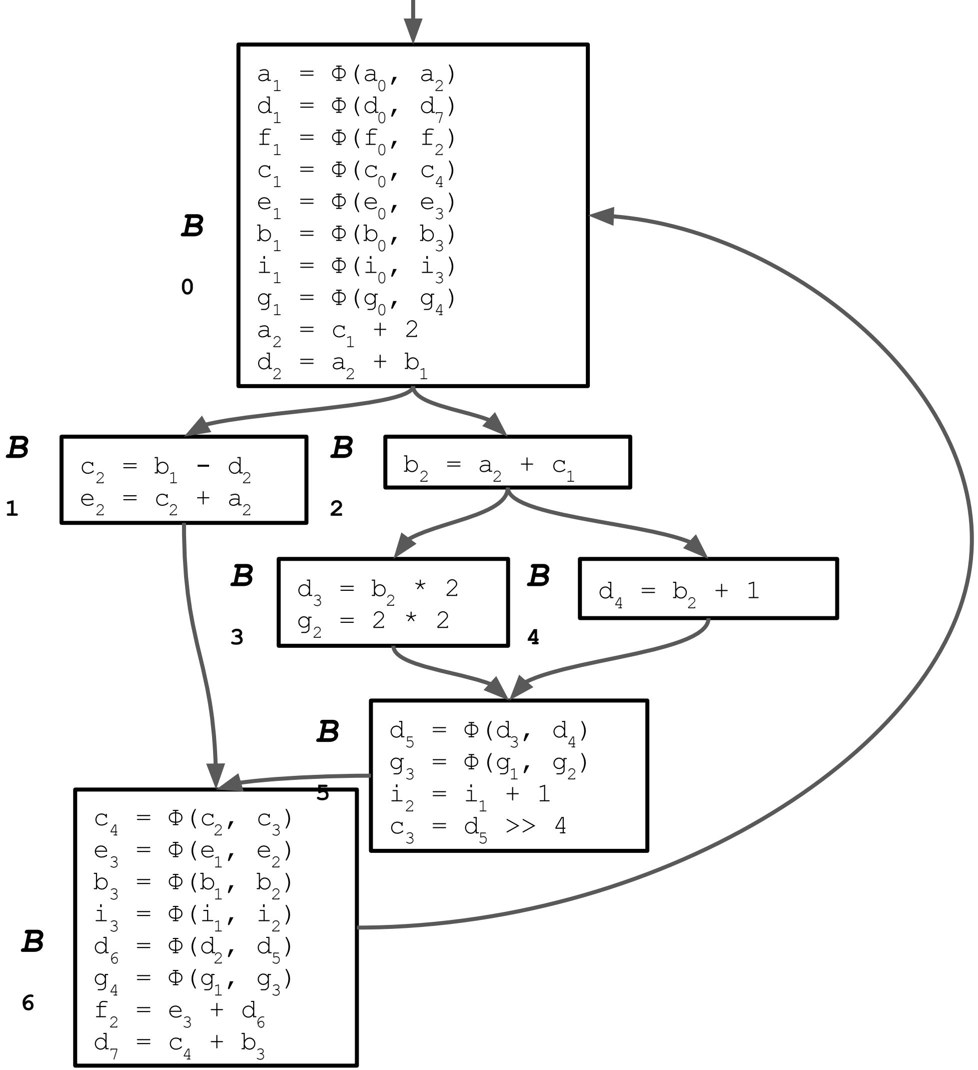

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5 | ∅ |
| 1 | 2, 3 | 5 |
| 2 | ∅ | 5 |
| 3 | ∅ | 5 |
| 4 | ∅ | 4, 5 |
| 5 | ∅ | ∅ |

# Problem 3



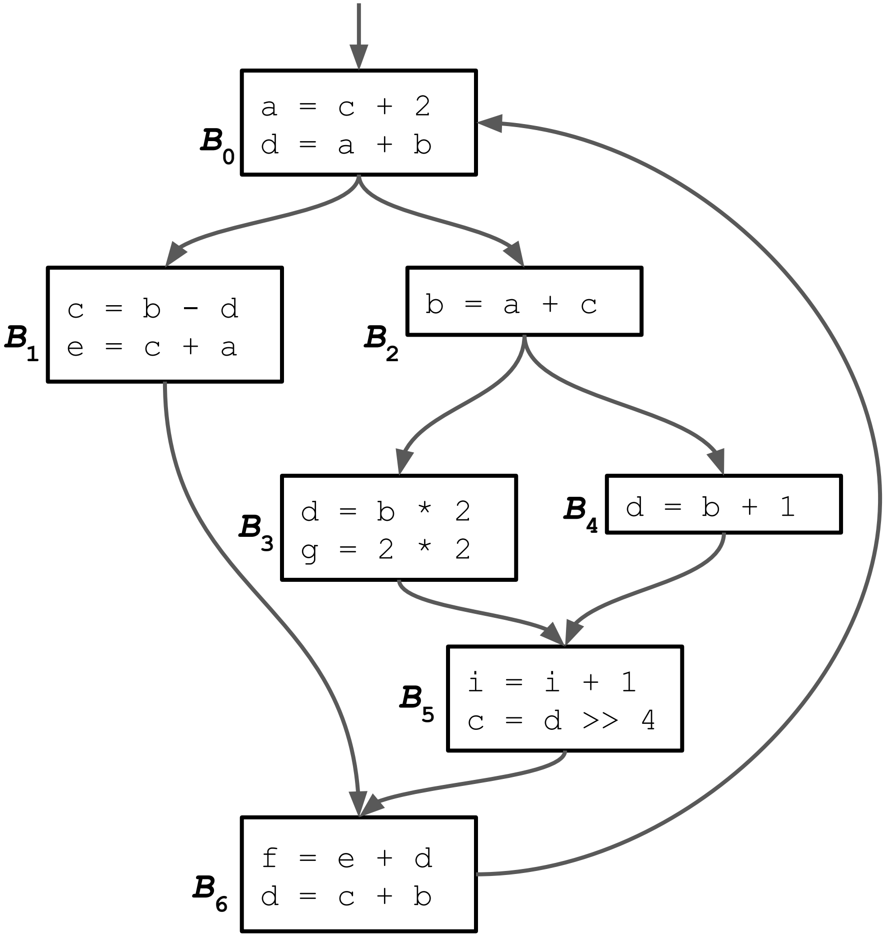
B0
a = c + 2
d = a + b
B1
c = b - d
e = c + a
B2
b = a + c
B3
d = b * 2
g = 2 * 2
B4
d = b + 1
B5
i = i + 1
c = d >> 4
B6
f = e + d
d = c + b

# Solution

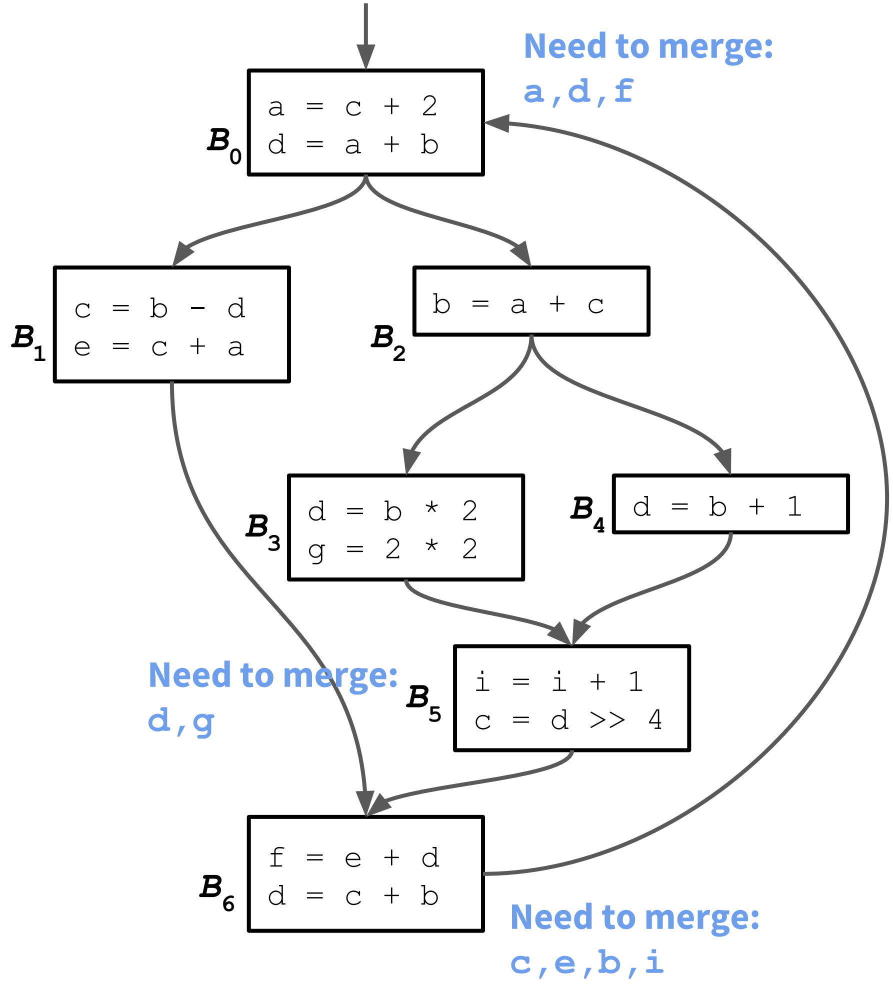


B
0
a1 = Φ(a0, a2)
d1 = Φ(d0, d7)
f1 = Φ(f0, f2)
c1 = Φ(c0, c4)
e1 = Φ(e0, e3)
b1 = Φ(b0, b3)
i1 = Φ(i0, i3)
g1 = Φ(g0, g4)
a2 = c1 + 2
d2 = a2 + b1
B
1
c2 = b1 - d2
e2 = c2 + a2
B
2
b2 = a2 + c1
B
3
d3 = b2 * 2
g2 = 2 * 2
B
4
d4 = b2 + 1
B
5
d5 = Φ(d3, d4)
g3 = Φ(g1, g2)
i2 = i1 + 1
c3 = d5 >> 4
B
6
c4 = Φ(c2, c3)
e3 = Φ(e1, e2)
b3 = Φ(b1, b2)
i3 = Φ(i1, i2)
d6 = Φ(d2, d5)
g4 = Φ(g1, g3)
f2 = e3 + d6
d7 = c4 + b3

# **Step 1**: Compute Dominance Frontiers



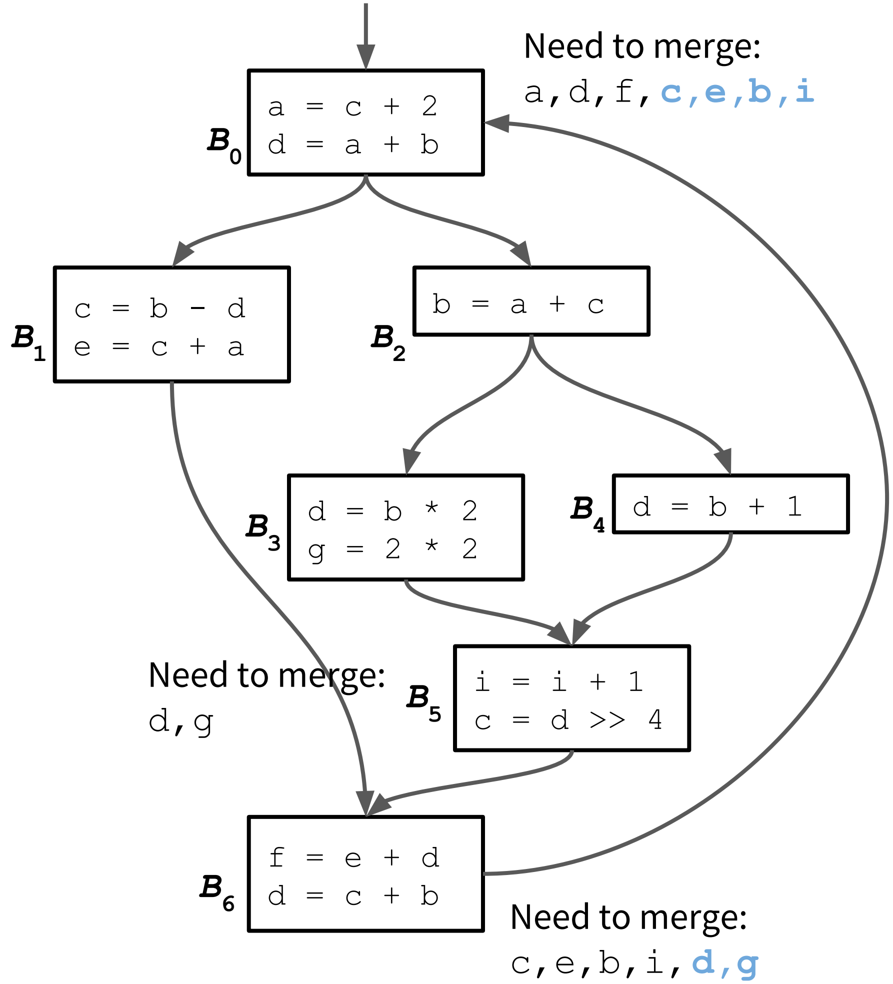

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5, 6 | 0 |
| 1 | ∅ | 6 |
| 2 | 3, 4, 5 | 6 |
| 3 | ∅ | 5 |
| 4 | ∅ | 5 |
| 5 | ∅ | 6 |
| 6 | ∅ | 0 |




# **Step 2**: Determine Necessary Merges

Each node in the dominance frontier of node X will merge definitions created in node X

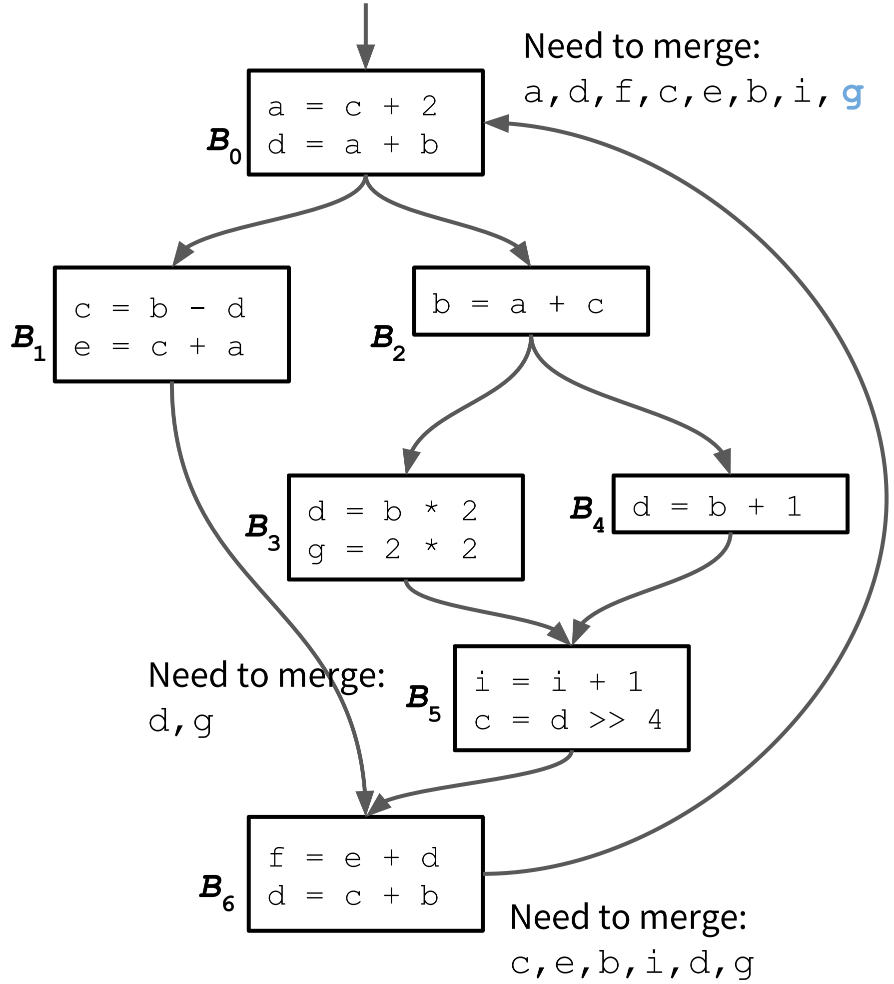
| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5, 6 | 0 |
| 1 | ∅ | 6 |
| 2 | 3, 4, 5 | 6 |
| 3 | ∅ | 5 |
| 4 | ∅ | 5 |
| 5 | ∅ | 6 |
| 6 | ∅ | 0 |




# **Step 3**: Continue Computing Merges

Each merge will create a new definition, and that definition may need to be merged again -- continue until there are no changes

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3,<br>4, 5, 6 | 0 |
| 1 | ∅ | 6 |
| 2 | 3, 4, 5 | 6 |
| 3 | ∅ | 5 |
| 4 | ∅ | 5 |
| 5 | ∅ | 6 |
| 6 | ∅ | 0 |




# **Step 3**: Continue Computing Merges

Each merge will create a new definition, and that definition may need to be merged again -- continue until there are no changes

| NODE | STRICTLY DOMINATES | DOMINANCE FRONTIER |
|---|---|---|
| 0 | 1, 2, 3, 4, 5, 6 | 0 |
| 1 | ∅ | 6 |
| 2 | 3, 4, 5 | 6 |
| 3 | ∅ | 5 |
| 4 | ∅ | 5 |
| 5 | ∅ | 6 |
| 6 | ∅ | 0 |

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_0$

```
a = c + 2
d = a + b
```

Need to merge:

`a,d,f,c,e,b,i,g`

$B_0$

$$
\begin{aligned}
a_1 &= \Phi(a_0,\ a_2)\\
d_1 &= \Phi(d_0,\ d_7)\\
f_1 &= \Phi(f_0,\ f_2)\\
c_1 &= \Phi(c_0,\ c_4)\\
e_1 &= \Phi(e_0,\ e_3)\\
b_1 &= \Phi(b_0,\ b_3)\\
i_1 &= \Phi(i_0,\ i_3)\\
g_1 &= \Phi(g_0,\ g_4)\\
a_2 &= c_1 + 2\\
d_2 &= a_2 + b_1
\end{aligned}
$$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_1$

```
c = b - d
e = c + a
```

Nothing to merge

$\Rightarrow$

$B_1$

$$c_2 = b_1 - d_2$$
$$e_2 = c_2 + a_2$$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_2$ `b = a + c`

Nothing to merge

$B_2$ $b_2 = a_2 + c_1$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_3$

```
d = b * 2
g = 2 * 2
```

Nothing to merge

$B_3$

$d_3 = b_2 * 2$

$g_2 = 2 * 2$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_4$: `d = b + 1`

Nothing to merge

$B_4$: $d_4 = b_2 + 1$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_5$

```
i = i + 1
c = d >> 4
```

Need to merge:

`d, g`

$\Rightarrow$

$B_5$

$$
\begin{aligned}
d_5 &= \Phi(d_3,\ d_4) \\
g_3 &= \Phi(g_1,\ g_2) \\
i_2 &= i_1 + 1 \\
c_3 &= d_5 \gg 4
\end{aligned}
$$

## **Step 4**: Write SSA Definitions

Merges go first, and each successive definition of a variable should increment its index by 1.

$B_6$

```
f = e + d
d = c + b
```

Need to merge:

`c,e,b,i,d,g`

$B_6$

$$
\begin{aligned}
c_4 &= \Phi(c_2, c_3) \\
e_3 &= \Phi(e_1, e_2) \\
b_3 &= \Phi(b_1, b_2) \\
i_3 &= \Phi(i_1, i_2) \\
d_6 &= \Phi(d_2, d_5) \\
g_4 &= \Phi(g_1, g_3) \\
f_2 &= e_3 + d_6 \\
d_7 &= c_4 + b_3
\end{aligned}
$$

# Thanks for a Great Quarter!

- The 401 18au Staff :)