Homework # 3 Solutions

CSE 403

February 25, 1999

1. RTL: Alarm Clock

   First, everytime AC occurs, the BELL starts simultaneously. Now, there are two cases where the bell will stop. The bell can stop in two case either 30 seconds goes by and STOP has not occured, or, BELL stops when STOP occurs and the STOP needs to be the first STOP after AC goes occurs.

   $$\forall_{w,t_{\mathrm{ac}}} \neg \exists_x \exists_{y,z}\ @(\mathrm{AC}, w) = @(\uparrow \mathrm{BELL}, x) = t_{\mathrm{ac}} \wedge$$
   $$((@(\downarrow \mathrm{BELL}, y) = t_{\mathrm{ac}} + 30 \wedge t_{\mathrm{ac}} \leq @(\mathrm{STOP}, x) \leq t_{\mathrm{ac}} + 30)$$
   $$\vee (@(\downarrow \mathrm{BELL}, x) = @(\mathrm{STOP}, z) \wedge @(\mathrm{STOP}, z) < t_{\mathrm{ac}} + 30 \wedge @(\mathrm{STOP}, z-1) < t_{\mathrm{ac}}))$$

2. RTL: Reader, Writer

   Two properties to express:

   (a) There is always a "Read" between consecutive "Write"

   $$\forall_i\ \exists_j (@(W,i) < @(R,j) < @(W,i+1))$$
   $$\neg \exists_j (@(W,i) = @(R,j))$$

   (b) If a read occurs at t_1, any "Read" or "Write" happens 2 time units after the "Read"

   $$\forall_{i,t_1}\ (@(R,i) = t_1)$$
   $$\forall_{j,t_2} (@(R,j) = t_2 \wedge j < i \rightarrow t_1 \geq t_2 + 2)$$
   $$\forall_{j,t_2} (@(W,j) = t_2 \wedge t_1 < t_2 \rightarrow t_2 \geq t_1 + 2)$$

3. RTL: Gate Controller

$\forall_i \ (@(\mathrm{cg}, i) = t_1) \rightarrow$

$\quad \exists_j (@(\mathrm{cc}, j) = t_2 \wedge t_1 < t_2 \leq t_1 + 5) \wedge \neg \exists_k (@(\mathrm{og}, k) = t_3 \wedge t_1 < t_3 < t_2)$

$\vee \ \exists_j (@(\mathrm{og}, j) = t_4 \wedge t_1 < t_4 \leq t_1 + 5) \wedge \neg \exists_k (@(\mathrm{cc}, k) = t_5 \wedge t_1 < t_5 < t_4)$

4. Algebraic Specification: Stacks

Bottom(NewStack) = Error

Bottom(Insert(Newstack, i)) = NewStack

Bottom(Insert(S, i)) = Bottom(S)

5. Algebraic Specifcation: Sets

   (a) Cardinality

   Cardinality(NewSet) = 0

   Cardinality(Insert(S, i)) = 1 + Cardinality(Delete(S, i))

   (b) Difference

   Difference(S, NewSet) = S

   Difference(S1, Insert(S,i)) = Difference(Delete(S1, i), S)

6. Algebraic Specification: Strings

Reverse(New()) = New()

Reverse(add(s,c)) = Append(Add(New(),c), Reverse(s))

7. Algebraic Specification: Bags

   (a) Delete

   Delete(NewBag, i) = NewBag

   Delete(Insert(B, i), i) = B

   Delete(Insert(B, i), j) = Insert(Delete(B, j), i)

   (b) Count

   Count(NewBag, i) = 0

   Count(Insert(B, i), i) = 1 + Count(B, i)

   Count(Insert(B, i), j) = Count(B, j))