CSE 413 Au12 Midterm Sample Solution

Question 1. (20 points) (programming warmup) Write a Racket function `sumof` that computes the sum of the integers in its list argument. The elements of the list can be simple integers, or nested lists that contain integers or other nested lists. Examples:

```
(sumof '(1 2 3)) => 6
(sumof '(1 (2 (3 4) 5))) => 15
(sumof '((((42))))) => 42
```

Your solution does not need to be tail-recursive. You may define additional helper functions at top-level if you really need them.

You should assume that the only elements of the lists are integers or nested lists. You do not need to deal with other data types.

Hint: Racket has functions to test types of expressions including `number? pair?`

```
(define (sumof lst)
  (cond ((null? lst) 0)
        ((number? (car lst)) (+ (car lst) (sumof (cdr lst))))
        (else (+ (sumof (car lst)) (sumof (cdr lst))))))
```

An equally good solution would be to test `(pair? lst)` instead of `(number? lst)` and switch the two recursive cases.

Question 2. (20 points) Write a tail recursive function `(minpower x target)` that, given two positive integers `x` and `target`, returns the smallest power of `x` that is at least as large as `target`. i.e., the result is x or x^2 or …, or x^n such that x^n >= `target` and no smaller power of x is at least that large. Examples:

`(minpower 2 12)` => 16 (since $2^3 = 8$ and $2^4 = 16$, which is the smallest power >= 12)
`(minpower 3 9)` => 9 (since $3^2 = 9$)

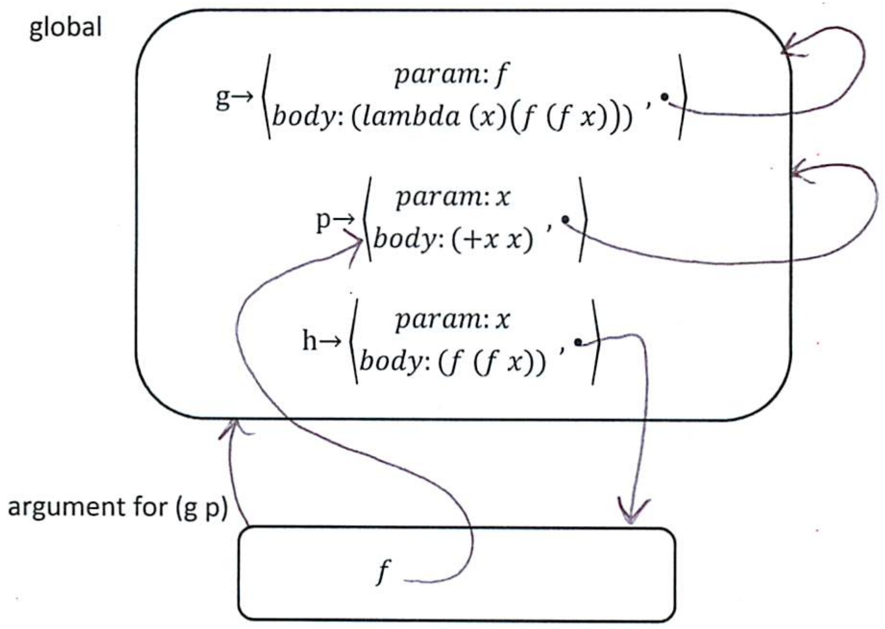
You may define any auxiliary functions you need at top level – they don't need to be nested inside `minpower` using `letrec` or anything similar (although you can do that if you want). For full credit your function must be properly tail recursive.

```
(define (minpower x target)
  (minaux x x target))


; if acc >= target return acc else return smallest
; value such that acc*x^n >= target (for some n>0)

(define (minaux acc x target)
  (if (>= acc target)
      acc
      (minaux (* acc x) x target)))
```

Question 3. (20 points) Pictures! Consider the following Racket definitions entered at the top-level of the Racket interpreter.

```
(define g (lambda (f) (lambda (x) (f (f x)))))
(define p (lambda (x) (+ x x)))
(define h (g p))
```

(a) Draw a diagram showing the environments, bindings, and closures created by the above definitions. Then answer part (b) below.



global
g→ ⟨ param: f, body: (lambda (x)(f (f x))) , • ⟩
p→ ⟨ param: x, body: (+x x) , • ⟩
h→ ⟨ param: x, body: (f (f x)) , • ⟩
argument for (g p)
f

Note: f in the argument environment for (g p) refers to the same closure as p in the global environment. Technically both are bound to the same closure, but it would also be fine to copy the closure to simplify the diagram.

(b) What is the value of `(h 3)` if we evaluate it after evaluating the above definitions?

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Question 4. (20 points) In homework 5 we implemented a memoized version of the `(comb n k)` function, and we saw a memoized version of the Fibonacci function in lecture. For this problem, implement a memoized version of factorial. `(fact n)` should return n! (= 1 * 2 * 3 * … * n), except that it should retain previously computed values and reuse those values to avoid recomputing previously calculated answers.

For this problem it's fine to store the values in an association list. An association list is a list of pairs. The function `assoc` can be used to retrieve values. For example, if `lst` is `(cons (cons 1 2) (cons (cons 3 4) null))`, then `(assoc 1 lst)` is `'(1 . 2)` and `(assoc 17 lst)` is `#f`.

Complete the following code to implement the memoized factorial function. Your code may not define any additional top-level functions.

```
(define fact
  (letrec ((memo null)
           (f (lambda (n)
                (let ((ans (assoc n memo)))
                  (if ans
                      (cdr ans)
                      (let ((calc-ans (if (< n 2)
                                          1
                                          (* n (f (- n 1))))))
                        (begin
                          (set! memo (cons (cons n calc-ans) memo))
                          calc-ans)))))))
    f))
```

Question 5. (20 points) We'd like to add a feature to MUPL to produce E-MUPL (enhanced MUPL). The new feature is an operation to test whether a MUPL expression evaluates to a MUPL int.

(DON'T PANIC!!! The answer is considerably shorter than the question!)

Here is the specification for the new MUPL `isint` expression:

- If *e* is a MUPL expression, then `(isint e)` is a MUPL expression. The value of `(isint e)` is either the MUPL value `(int 1)` if evaluating `e` produces a MUPL int, or is the MUPL value `(int 0)` if `e` is some other kind of MUPL expression (apair, closure, etc.). In other words, it is similar to the existing `isaunit` function that evaluates to 1 or 0 depending on whether its operand is a MUPL `aunit`.

On the next page, write the code needed to add this new expression to the MUPL interpreter `eval-prog` function.

You should assume that the following structure has been added to MUPL to represent this expression:

```
(struct isint (e)) ;; evaluate to 1 if e is an int else 0
```

(the `#:transparent` directives have been omitted from the struct declarations in this problem to save space, but that does not change the meaning or use of the struct data types.)

For reference, here are the other structures defined in the original MUPL code (most of which you probably won't need):

```
(struct var  (string))  ;; a variable, e.g., (var "foo")
(struct int  (num)    )  ;; a constant number, e.g., (int 17)
(struct add  (e1 e2) )  ;; add two expressions
(struct ifgreater (e1 e2 e3 e4)    ) ;; if e1 > e2 then e3 else e4
(struct fun  (nameopt formal body)) ;; a recursive(?) 1-argument function
(struct call (funexp actual)       ) ;; function call
(struct mlet (var e body)) ;; a local binding (let var = e in body)
(struct apair (e1 e2)     ) ;; make a new pair
(struct fst  (e)    ) ;; get first part of a pair
(struct snd  (e)    ) ;; get second part of a pair
(struct aunit ()    ) ;; unit value -- good for ending a list
(struct isaunit (e)) ;; evaluate to 1 if e is aunit else 0

;; a closure is not in "source" programs; it's what functions evaluate to
(struct closure (env fun))
```

Reminder: the Racket function `(error "`*message*`")` can be used to terminate evaluation with the given message.

Write your code on the next page. (You can tear this page out of the exam for reference if that is convenient.)

Question 5. (cont.) Write your code to implement the new MUPL `isint` expression below.

```
(struct isint (e)) ;; evaluate to 1 if e is an int else 0

(define (eval-prog p)
  (letrec
    ((f (lambda (env p)
          (cond (...
                 ((isint? p)  ;; write your code for isint below
                  (

                   (let ([v (f env (isint-e p))])

                     (if (int? v) (int 1) (int 0)))

                  )
           )
           ...
           ;; remainder of eval-prog omitted
```