CSE 417
Introduction to Algorithms
Winter 2007

NP-Completeness

(Chapter 8)

Some Algebra Problems (Algorithmic)

Given positive integers a, b, c

Question 1: does there exist a positive integer x such that ax = c ?

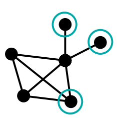
Question 2: does there exist a positive integer x such that $ax^2 + bx = c$?

Question 3: do there exist positive integers x and y such that $ax^2 + by = c$?

Some Problems

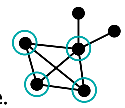
Independent-Set:

Given a graph G=(V,E) and an integer k, is there a subset U of V with |U| ≥ k such that no two vertices in U are joined by an edge.



Clique:

Given a graph G=(V,E) and an integer k, is there a subset U of V with |U| ≥ k such that every pair of vertices in U is joined by an edge.



A Brief History of Ideas

From Classical Greece, if not earlier, "logical thought" held to be a somewhat mystical ability

Mid 1800's: Boolean Algebra and foundations of mathematical logic created possible "mechanical" underpinnings

1900: David Hilbert's famous speech outlines program: mechanize all of mathematics?
http://mathworld.wolfram.com/HilbertsProblems.html

1930's: Gödel, Church, Turing, et al. prove it's impossible

More History

1930/40's

- What is (is not) computable

1960/70's

- What is (is not) feasibly computable
- Goal – a (largely) technology-independent theory of time required by algorithms
- Key modeling assumptions/approximations
  - Asymptotic (Big-O), worst case is revealing
  - Polynomial, exponential time – qualitatively different

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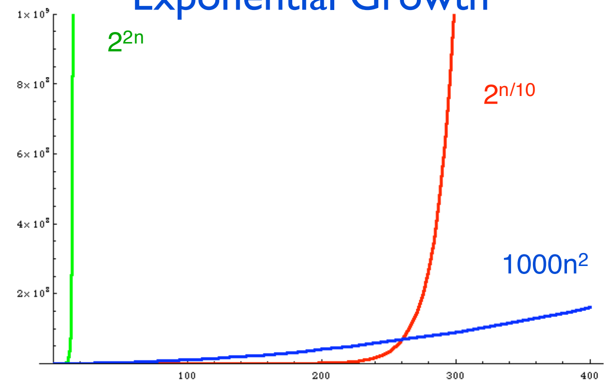
Polynomial vs Exponential Growth



Another view of Poly vs Exp

Next year's computer will be 2x faster. If I can solve problem of size n_0 today, how large a problem can I solve in the same time next year?

Complexity	Increase	E.g. $T=10^{12}$	
$O(n)$	$n_0 \rightarrow 2n_0$	10^{12}	2×10^{12}
$O(n^2)$	$n_0 \rightarrow \sqrt{2}\, n_0$	10^6	1.4×10^6
$O(n^3)$	$n_0 \rightarrow \sqrt[3]{2}\, n_0$	10^4	1.25×10^4
$2^{n/10}$	$n_0 \rightarrow n_0+10$	400	410
2^n	$n_0 \rightarrow n_0+1$	40	41

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Polynomial versus exponential

We'll say any algorithm whose run-time is

- polynomial is good
- bigger than polynomial is bad

Note – of course there are exceptions:

- n^{100} is bigger than $(1.001)^n$ for most practical values of n but usually such run-times don't show up
- There are algorithms that have run-times like $O(2^{\mathrm{sqrt}(n)/22})$ and these may be useful for small input sizes, but they're not too common either

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Some Convenient Technicalities

"Problem" – the general case

Ex: The Clique Problem: Given a graph G and an integer k, does G contain a k-clique?

"Problem Instance" – the specific cases

Ex: Does  contain a 4-clique? (no)

Ex: Does  contain a 3-clique? (yes)

Decision Problems – Just Yes/No answer

Problems as Sets of "Yes" Instances

Ex: CLIQUE = { (G,k) | G contains a k-clique }

E.g., ( , 4) $\notin$ CLIQUE

E.g., ( , 3) $\in$ CLIQUE

Decision problems

Computational complexity usually analyzed using decision problems

answer is just 1 or 0 (yes or no).

Why?

much simpler to deal with

deciding whether G has a k-clique, is certainly no harder than finding a k-clique in G, so a lower bound on deciding is also a lower bound on finding

Less important, but if you have a good decider, you can often use it to get a good finder. (Ex.: does G still have a k-clique after I remove this vertex?)

The class P

Definition: P = set of (decision) problems solvable by computers in polynomial time. i.e.,

$T(n) = O(n^k)$ for some fixed k.

These problems are sometimes called tractable problems.

Examples: sorting, shortest path, MST, connectivity, RNA folding & other dyn. prog. – most of 417

(exceptions: Change-Making/Stamps, TSP)

Beyond P?

There are many natural, practical problems for which we don't know any polynomial-time algorithms

e.g. CLIQUE:

Given an undirected graph G and an integer k, does G contain a k-clique?

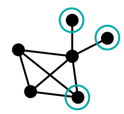
e.g. quadratic Diophantine equations:

Given a, b, c $\in$ N, $\exists$ x, y $\in$ N s.t. $ax^2 + by = c$?

Some Problems

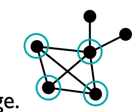
Independent-Set:

Given a graph G=(V,E) and an integer k, is there a subset U of V with |U| ≥ k such that no two vertices in U are joined by an edge.



Clique:

Given a graph G=(V,E) and an integer k, is there a subset U of V with |U| ≥ k such that every pair of vertices in U is joined by an edge.



Some More Problems

Euler Tour:

Given a graph G=(V,E) is there a cycle traversing each edge once.

Hamilton Tour:

Given a graph G=(V,E) is there a simple cycle of length |V|, i.e., traversing each vertex once.

TSP:

Given a weighted graph G=(V,E,w) and an integer k, is there a Hamilton tour of G with total weight ≤ k.

Satisfiability

Boolean variables $x_1, ..., x_n$

taking values in {0,1}. 0=false, 1=true

Literals

x_i or $\neg x_i$ for i = 1, ..., n

Clause

a logical OR of one or more literals

e.g. $(x_1 \vee \neg x_3 \vee x_7 \vee x_{12})$

CNF formula

a logical AND of a bunch of clauses

Satisfiability

CNF formula example

$(x_1 \vee \neg x_3 \vee x_7) \wedge (\neg x_1 \vee \neg x_4 \vee x_5 \vee \neg x_7)$

If there is some assignment of 0's and 1's to the variables that makes it true then we say the formula is satisfiable

the one above is, the following isn't

$x_1 \wedge (\neg x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge \neg x_3$

Satisfiability: Given a CNF formula F, is it satisfiable?

Satisfiable?

$$
\begin{aligned}
&(\ x \lor y \lor z\ ) \land (\neg x \lor y \lor \neg z\ ) \land \\
&(\ x \lor \neg y \lor z\ ) \land (\neg x \lor \neg y \lor z\ ) \land \\
&(\neg x \lor \neg y \lor \neg z\ ) \land (\ x \lor y \lor z\ ) \land \\
&(\ x \lor \neg y \lor z\ ) \land (\ x \lor y \lor \neg z\ )
\end{aligned}
$$

$$
\begin{aligned}
&(\ x \lor y \lor z\ ) \land (\neg x \lor y \lor \neg z\ ) \land \\
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&(\ x \lor \neg y \lor z\ ) \land (\ x \lor y \lor \neg z\ )
\end{aligned}
$$

More History – As of 1970

Many of the above problems had been studied for decades

All had real, practical applications

None had poly time algorithms; exponential was best known

But, it turns out they all have a very deep similarity under the skin

Some Problem Pairs

Euler Tour	Hamilton Tour
2-SAT	3-SAT
Min Cut	Max Cut
Shortest Path	Longest Path

Superficially different; similar computationally

Similar pairs; seemingly different computationally

Common property of these problems

There is a special piece of information, a short hint or proof, that allows you to efficiently (in polynomial-time) verify that the YES answer is correct. *BUT*, this hint might be *very hard to find*

e.g.

TSP: the tour itself

Independent-Set, Clique: the vertex set U

Satisfiability: an assignment that makes formula true

Quadratic Diophantine eqns: the numbers x & y

The complexity class NP

NP consists of all decision problems where

You can verify the YES answers efficiently (in polynomial time) given a short (polynomial-size) hint

And

No hint can fool your polynomial time verifier into saying YES for a NO instance

(implausible for all exponential time problems)

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More Precise Definition of NP

A decision problem is in NP iff there is a polynomial time procedure v(-,-), and an integer k such that

for every YES problem instance x there is a hint h with $|h| \leq |x|^k$ such that v(x,h) = YES

and

for every NO problem instance x there is no hint h with $|h| \leq |x|^k$ such that v(x,h) = YES

"Hints" sometimes called "Certificates"

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Example: CLIQUE is in NP

procedure v(x,h)

if

x is a well-formed representation of a graph G = (V, E) and an integer k,

and

h is a well-formed representation of a k-vertex subset U of V,

and

U is a clique in G,

then output "YES"

else output "I'm unconvinced"

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Is it correct?

For every x = (G,k) such that G contains a k-clique, there is a hint h that will cause v(x,h) to say YES, namely h = a list of the vertices in such a k-clique

and

No hint can fool v into saying yes if either x isn't well-formed (the uninteresting case) or if x = (G,k) but G does not have any cliques of size k (the interesting case)

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Another example: SAT ∈ NP

Hint: the satisfying assignment A

Verifier: v(F,A) = syntax(F,A) && satisfies(F,A)

- Syntax: True iff F is a well-formed formula & A is a truth-assignment to its variables
- Satisfies: plug A into F and evaluate

Correctness:

- If F is satisfiable, it has some satisfying assignment A, and we'll recognize it
- If F is unsatisfiable, it doesn't, and we won't be fooled

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Keys to showing that a problem is in NP

What's the output? (must be YES/NO)

What's the input? Which are YES?

For every given YES input, is there a hint that would help? Is it polynomial length?

- OK if some inputs need no hint

For any given NO input, is there a hint that would trick you?

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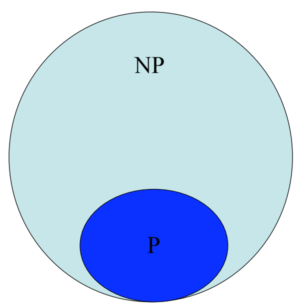
Complexity Classes

NP = Polynomial-time **verifiable**

P = Polynomial-time **solvable**



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Solving NP problems without hints

The most obvious algorithm for most of these problems is brute force:

- try all possible hints; check each one to see if it works.
- Exponential time:
  - 2^n truth assignments for n variables
  - n! possible TSP tours of n vertices
  - $\binom{n}{k}$ possible k element subsets of n vertices
  - etc.

…and to date, every alg, even much less-obvious ones, are slow, too

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Problems in P can also be verified in polynomial-time

Short Path: Given a graph G with edge lengths, is there a path from s to t of length ≤ k?

- Verify: Given a purported path from s to t, is it a path, is its length ≤ k?

Small Spanning Tree: Given a weighted undirected graph G, is there a spanning tree of weight ≤ k?

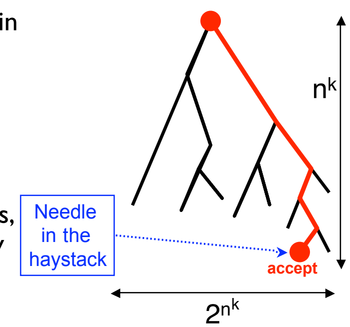
- Verify: Given a purported spanning tree, is it a spanning tree, is its weight ≤ k?

(But the hints aren't really needed in these cases…)

P vs NP vs Exponential Time

Theorem: Every problem in NP can be solved deterministically in exponential time

Proof: "hints" are only n^k long; try all 2^{n^k} possibilities, say by backtracking. If any succeed, say YES; if all fail, say NO.



P and NP

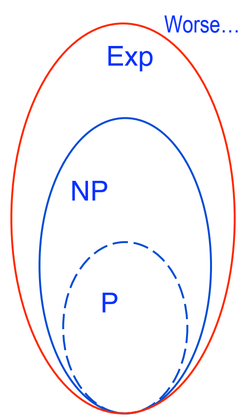
Every problem in P is in NP

- one doesn't even need a hint for problems in P so just ignore any hint you are given

Every problem in NP is in exponential time

I.e., P ⊆ NP ⊆ Exp

We know P ≠ Exp, so either P ≠NP, or NP ≠ Exp (most likely both)



P vs NP

Theory

- P = NP ?
- Open Problem!
- I bet against it

Practice

- Many interesting, useful, natural, well-studied problems known to be NP-complete
- With rare exceptions, no one routinely succeeds in finding exact solutions to large, arbitrary instances

A problem NOT in NP; A bogus "proof" to the contrary

EEXP = {(p,x) | prog p accepts input x in < $2^{2^{|x|}}$ steps }

NON Theorem: EEXP in NP

"Proof" I: Hint = step-by-step trace of the computation of p on x; verify step-by-step

"Proof" II: Hint = a bit; accept iff it's 1

More Connections

Some Examples in NP
- Satisfiability
- Independent-Set
- Clique
- Vertex Cover

All hard to solve; hints seem to help on all

Very surprising fact:

Fast solution to any gives fast solution to all!

NP-complete Problems

We are pretty sure that no problem in NP – P can be solved in polynomial time.

Non-Definition: NP-complete = the hardest problems in the class NP. (Formal definition later.)

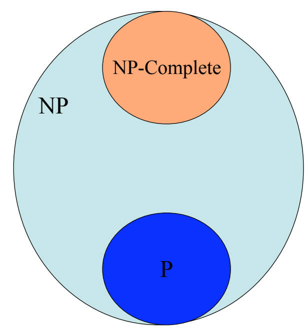
Interesting fact: If any one NP-complete problem could be solved in polynomial time, then all NP problems could be solved in polynomial time.

Complexity Classes

NP = Poly-time **verifiable**

P = Poly-time **solvable**

NP-Complete =
"Hardest" problems in **NP**



The class NP-complete (cont.)

Thousands of important problems have been shown to be NP-complete.

Fact (Dogma): The general belief is that there is no efficient algorithm for any NP-complete problem, but no proof of that belief is known.

Examples: SAT, clique, vertex cover, Hamiltonian cycle, TSP, bin packing.

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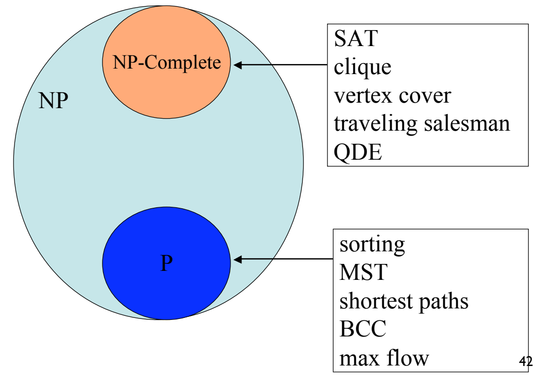
Complexity Classes of Problems



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Does P = NP?

This is an open question.

To show that P = NP, we have to show that every problem that belongs to NP can be solved by a polynomial time deterministic algorithm.

No one has shown this yet.

(It seems unlikely to be true.)

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Is all of this useful for anything?

Earlier in this class we learned techniques for solving problems in P.

Question: Do we just throw up our hands if we come across a problem we suspect not to be in P?

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Dealing with NP-complete Problems

What if I think my problem is not in P?

Here is what you might do:

1) Prove your problem is NP-hard or -complete
 (a common, but not guaranteed outcome)

2) Come up with an algorithm to solve the problem usually or approximately.

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Reductions: a useful tool

Definition: To reduce A to B means to solve A, given a subroutine solving B.

Example: reduce MEDIAN to SORT
- Solution: sort, then select (n/2)nd

Example: reduce SORT to FIND_MAX
- Solution: FIND_MAX, remove it, repeat

Example: reduce MEDIAN to FIND_MAX
- Solution: transitivity: compose solutions above.

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Reductions: Why useful

Definition: To reduce A to B means to solve A, given a subroutine solving B.

Fast algorithm for B implies fast algorithm for A
- (nearly as fast; takes some time to set up call, etc.)

If every algorithm for A is slow, then no algorithm for B can be fast.

“complexity of A” ≤ *“complexity of B” + “complexity of reduction”*

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SAT is NP-complete

Cook’s theorem: SAT is NP-complete

Satisfiability (SAT)
A Boolean formula in conjunctive normal form (CNF) is satisfiable if there exists a truth assignment of 0’s and 1’s to its variables such that the value of the expression is 1.
Example:
$S=(x+y+\neg z)\bullet(\neg x+y+z)\bullet(\neg x+\neg y+\neg z)$
Example above is satisfiable. (We can see this by setting $x=1$, $y=1$ and $z=0$.)

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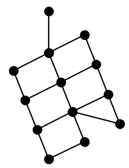
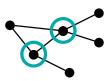
NP-complete problem: Vertex Cover

Input: Undirected graph G = (V, E), integer k.

Output: True iff there is a subset C of V of size ≤ k such that every edge in E is incident to at least one vertex in C.

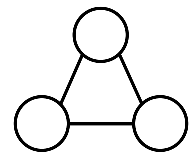


Example: Vertex cover of size ≤ 2.

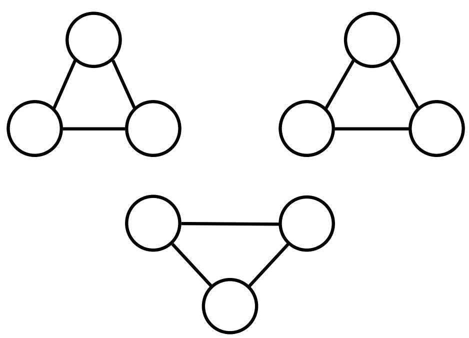


In NP? Exercise

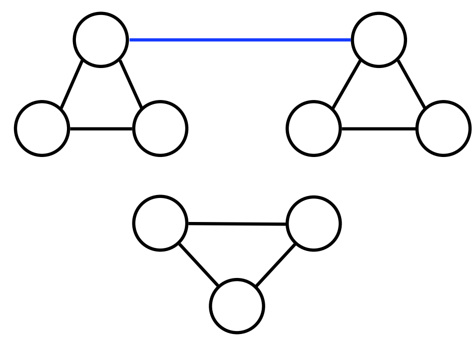
3SAT $\le_p$ VertexCover



3SAT $\le_p$ VertexCover

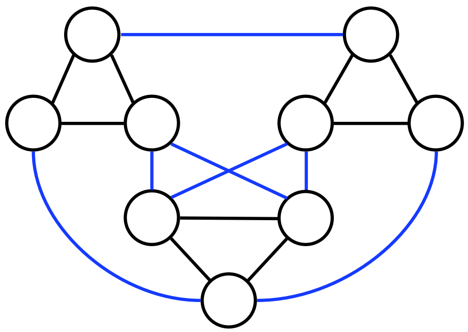


3SAT $\le_p$ VertexCover



3SAT $\leq_p$ VertexCover

k=6



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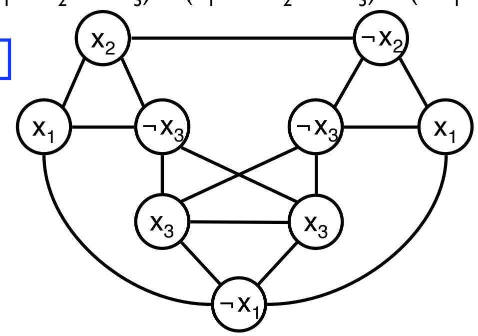
3SAT $\leq_p$ VertexCover

$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3)$

$\boxed{k=6}$



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3SAT $\leq_p$ VertexCover

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3-SAT Instance:

- Variables: $x_1, x_2, \ldots$
- Literals: $y_{i,j}$, $1 \leq i \leq q$, $1 \leq j \leq 3$
- Clauses: $c_i = y_{i1} \vee y_{i2} \vee y_{i3}$, $1 \leq l \leq q$
- Formula: $c = c_1 \wedge c_2 \wedge \ldots \wedge c_q$

) =

VertexCover Instance:

- $k = 2q$
- $G = (V, E)$
- $V = \{ [i,j] \mid 1 \leq i \leq q, 1 \leq j \leq 3 \}$
- $E = \{ ([i,j], [k,l]) \mid i = k \text{ or } y_{ij} = \neg y_{kl} \}$

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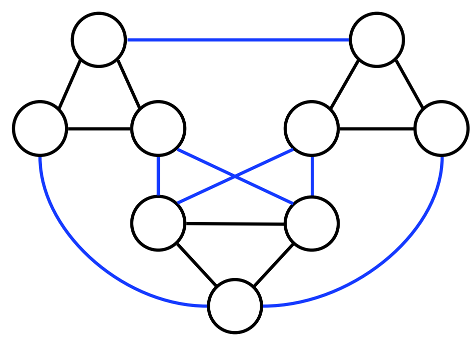
3SAT $\leq_p$ VertexCover

k=6



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Correctness of “3SAT $\leq_p$ VertexCover”

Summary of reduction function f: Given formula, make graph G with one group per clause, one node per literal. Connect each to all nodes in same group, plus complementary literals (x, ¬x). Output graph G plus integer k = 2 * number of clauses. Note: *f does not know whether formula is satisfiable or not; does not know if G has k-cover; does not try to find satisfying assignment or cover.*

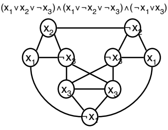
Correctness:

Show f poly time computable: A key point is that graph size is polynomial in formula size; mapping basically straightforward.

Show c in 3-SAT iff f(c)=(G,k) in VertexCover:
(⇒) Given an assignment satisfying c, pick one true literal per clause. Add other 2 nodes of each triangle to cover. Show it is a cover: 2 per triangle cover triangle edges; only true literals (but perhaps not all true literals) uncovered, so at least one end of every (x, ¬x) edge is covered.
(⇐) Given a k-vertex cover in G, uncovered labels define a valid (perhaps partial) truth assignment since no (x, ¬x) pair uncovered. It satisfies c since there is one uncovered node in each clause triangle (else some other clause triangle has > 1 uncovered node, hence an uncovered edge.)

Utility of “3SAT $\leq_p$ VertexCover”

$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3)$



Suppose we had a fast algorithm for VertexCover, then we could get a fast algorithm for 3SAT:

Given 3-CNF formula w, build Vertex Cover instance y = f(w) as above, run the fast VC alg on y; say “YES, w is satisfiable” iff VC alg says “YES, y has a vertex cover of the given size”

On the other hand, suppose no fast alg is possible for 3SAT, then we know none is possible for VertexCover either.

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“3SAT $\leq_p$ VertexCover” Retrospective

Previous slide: two suppositions

Somewhat clumsy to have to state things that way.

Alternative: abstract out the key elements, give it a name (“polynomial time reduction”), then properties like the above always hold.

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Polynomial-Time Reductions

Definition: Let A and B be two problems.

We say that A is *polynomially reducible* to B if there exists a polynomial-time algorithm f that converts each instance x of problem A to an instance f(x) of B such that

x is a YES instance of A iff
f(x) is a YES instance of B.

$x \in A \;\Leftrightarrow\; f(x) \in B$

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Polynomial-Time Reductions (cont.)

Define: $A \leq_p B$ "A is polynomial-time reducible to B", iff there is a polynomial-time computable function f such that: $x \in A \iff f(x) \in B$

Why the notation?

"complexity of A" ≤ "complexity of B" + "complexity of f"

polynomial

(1) $A \leq_p B$ and $B \in P \Rightarrow A \in P$

(2) $A \leq_p B$ and $A \notin P \Rightarrow B \notin P$

(3) $A \leq_p B$ and $B \leq_p C \Rightarrow A \leq_p C$ (transitivity)

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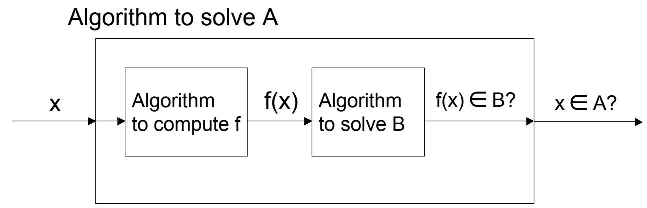
Using an Algorithm for *B* to Solve *A*



"If $A \leq_p B$, and we can solve B in polynomial time, then we can solve A in polynomial time also."

Ex: suppose f takes $O(n^3)$ and algorithm for B takes $O(n^2)$. How long does the above algorithm for A take?

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Definition of NP-Completeness

Definition: Problem B is NP-hard if every problem in NP is polynomially reducible to B.

Definition: Problem B is NP-complete if:

(1) B belongs to NP, and

(2) B is NP-hard.

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Proving a problem is NP-complete

Technically, for condition (2) we have to show that every problem in NP is reducible to B. (yikes!) This sounds like a lot of work.

For the very first NP-complete problem (SAT) this had to be proved directly.

However, once we have one NP-complete problem, then we don't have to do this every time.

Why? Transitivity.

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Re-stated Definition

Lemma: Problem B is NP-complete if:

(1) B belongs to NP, and

(2') A is polynomial-time reducible to B, for some problem A that is NP-complete.

That is, to show (2') given a new problem B, it is sufficient to show that SAT or any other NP-complete problem is polynomial-time reducible to B.

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Usefulness of Transitivity

Now we only have to show $L' \leq_p L$, for some NP-complete problem L', in order to show that L is NP-hard. Why is this equivalent?

1) Since L' is NP-complete, we know that L' is NP-hard. That is:

$\forall L'' \in NP$, we have $L'' \leq_p L'$

2) If we show $L' \leq_p L$, then by transitivity we know that: $\forall L'' \in NP$, we have $L'' \leq_p L$.

Thus L is NP-hard.

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Ex: VertexCover is NP-complete

3-SAT is NP-complete (shown by S. Cook)

3-SAT $\leq_p$ VertexCover

VertexCover is in NP (we showed this earlier)

Therefore VertexCover is also NP-complete

So, poly-time algorithm for VertexCover would give poly-time algs for everything in NP

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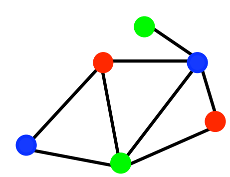
NP-complete problem: 3-Coloring

Input: An undirected graph G=(V,E).

Output: True iff there is an assignment of at most 3 colors to the vertices in G such that no two adjacent vertices have the same color.

Example:



In NP? Exercise

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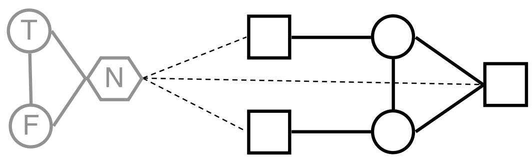
A 3-Coloring Gadget:

In what ways can this be 3-colored?



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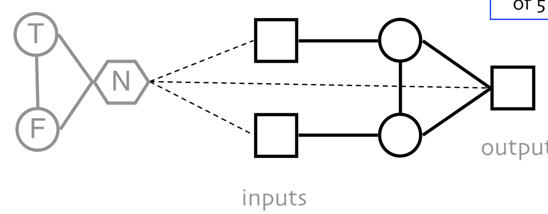
A 3-Coloring Gadget: "Sort of an OR gate"

if any input is T, the output can be T

if output is T, some input must be T

Exercise: find all colorings of 5 nodes



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3SAT $\leq_p$ 3Color

f (3-SAT Instance:
- Variables: $x_1, x_2, \ldots$
- Literals: $y_{i,j}$, $1 \leq i \leq q$, $1 \leq j \leq 3$
- Clauses: $c_i = y_{i1} \vee y_{i2} \vee y_{i3}$, $1 \leq l \leq q$
- Formula: $c = c_1 \wedge c_2 \wedge \ldots \wedge c_q$

) =

3Color Instance:
- G = (V, E)
- 6 q + 2 n + 3 vertices
- 13 q + 3 n + 3 edges
- (See Example for details)

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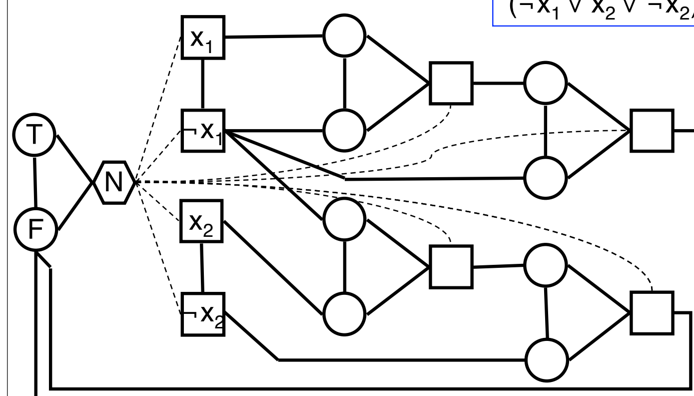
3SAT $\leq_p$ 3Color Example

$(x_1 \vee \neg x_1 \vee \neg x_1)$
$\wedge$
$(\neg x_1 \vee x_2 \vee \neg x_2)$



6 q + 2 n + 3 vertices   13 q + 3 n + 3 edges

Correctness of “3SAT $\leq_p$ 3Coloring”

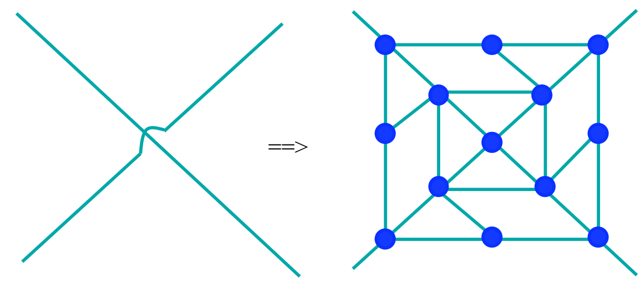
Summary of reduction function f:
Given formula, make G with T-F-N triangle, 1 pair of literal nodes per variable, 2 “or” gadgets per clause, connected as in example.
Note: again, f does not know or construct satisfying assignment or coloring.

Correctness:

Show f poly time computable: A key point is that graph size is polynomial in formula size; graph looks messy, but pattern is basically straightforward.

Show c in 3-SAT iff f(c) is 3-colorable:
($\Rightarrow$) Given an assignment satisfying c, color literals T/F as per assignment; can color “or” gadgets so output nodes are T since each clause is satisfied.
($\Leftarrow$) Given a 3-coloring of f(c), name colors T-N-F as in example. All square nodes are T or F (since all adjacent to N). Each variable pair (xi, ¬xi) must have complementary labels since they’re adjacent. Define assignment based on colors of xi’s. Clause “output” nodes must be colored T since they’re adjacent to both N & F. By fact noted earlier, output can be T only if at least one input is T, hence it is a satisfying assignment.

Planar 3-Coloring is also NP-Complete



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Common Errors in NP-completeness Proofs

Backwards reductions
Bipartiteness $\leq_p$ SAT is true, but not so useful.
(XYZ $\leq_p$ SAT shows XYZ in NP, does not show it’s hard.)

Sloooow Reductions
“Find a satisfying assignment, then output…”

Half Reductions
Delete dashed edges in 3Color reduction. It’s still true that “c satisfiable $\Rightarrow$ G is 3 colorable”, but 3-colorings don’t necessarily give good assignments.

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Coping with NP-Completeness

Is your real problem a special subcase?
- E.g. 3-SAT is NP-complete, but 2-SAT is not;
- Ditto 3- vs 2-coloring
- E.g. maybe you only need planar graphs, or degree 3 graphs, or …

Guaranteed approximation good enough?
- E.g. Euclidean TSP within 1.5 * Opt in poly time

Clever exhaustive search may be fast enough in practice, e.g. Backtrack, Branch & Bound, pruning

Heuristics – usually a good approximation and/or usually fast

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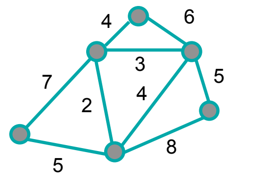
NP-complete problem: TSP

Input: An undirected graph G=(V,E) with integer edge weights, and an integer b.

Output: YES iff there is a simple cycle in G passing through all vertices (once), with total cost ≤ b.

Example:

b = 34



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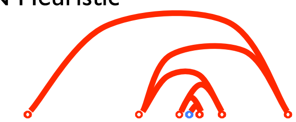
TSP - Nearest Neighbor Heuristic

Recall NN Heuristic



Fact: NN tour can be about (log n) x opt, i.e.

$$\lim_{n \to \infty} \frac{NN}{OPT} \to \infty$$

(above example is not that bad)

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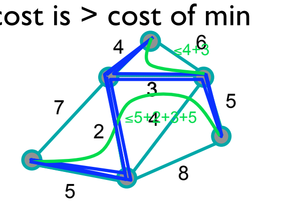
2x Approximation to EuclideanTSP

A TSP tour visits all vertices, so contains a spanning tree, so TSP cost is > cost of min spanning tree.

Find MST

Find "DFS" Tour

Shortcut



TSP ≤ shortcut < DFST = 2 * MST < 2 * TSP

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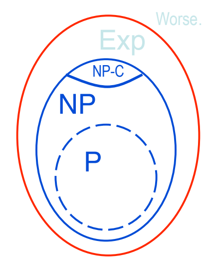
Summary



Big-O – good
P – good
Exp – bad
Exp, but hints help? NP
NP-hard, NP-complete – bad (I bet)
To show NP-complete – reductions
NP-complete = hopeless? – no, but you need to lower your expectations: heuristics & approximations.

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