P1) Draw out a maximum s-t flow for the graph below, and the corresponding residual graph G_f. What is the minimum cut that corresponds to this max flow?
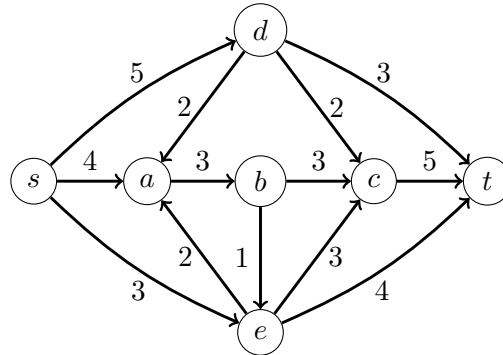
P2) Given an (unweighted) directed graph $G = (V, E)$, a pair of vertices s, t and an integer $1 \le k \le n$. Design an algorithm that runs in time polynomial in n, k and outputs yes if there are k *vertex* disjoint paths from s to t and no otherwise. For example, in the following graph there are two edge disjoint paths from s to t but no two vertex disjoint paths from s to t.
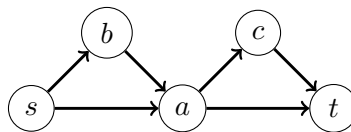
P3) Given a bipartite graph $G = (X, Y, E)$ with $|X| = |Y| = n$. Suppose the degree of every vertex of G is exactly k, for some integer $k \ge 1$. Design a polynomial time algorithm that colors edges of G with k colors such that any two edges that share an endpoint have distinct colors. For example in the following bipartite graph $k = 2$ and we have colored its edges with two colors.
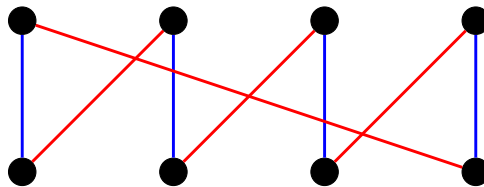
P4) **Extra Credit:** You are given an $m \times n$ array of real numbers. Suppose that the numbers in each row add up to an integer and the numbers in each column add up to an integer. You want to substitute each number $A[i, j]$ with $\lfloor A[i, j] \rfloor$ or $\lceil A[i, j] \rceil$ such that the sum of the numbers in

each row and each column remain invariant. Design a polynomial time algorithm that outputs the integer array.

For example, if the input is the left table you can output the right table. Note the sum of numbers in each row (and each column) of the left table is the same as the sum of the numbers of the same row (resp. the same column) in the right table.

0.4	0.1	1.5
0.6	1.9	0.5

$\Rightarrow$

1	0	1
0	2	1