CSE421: Design and Analysis of Algorithms	May 12, 2022
Homework 6	
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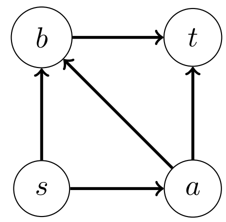
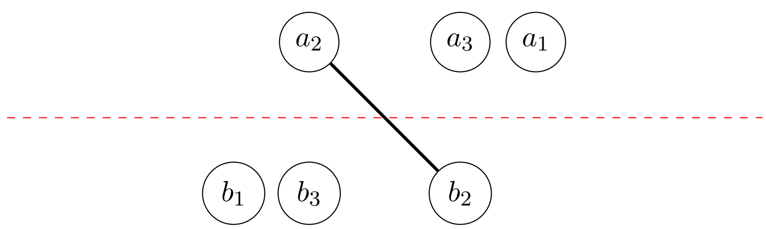
P1) Given a DAG with n vertices and m edges and a pair of vertices s, t. Design a polynomial time algorithm to output the number of directed paths from s to t in G. For example, given s, t in the following graph you should output 3.



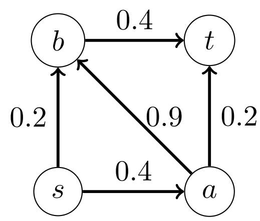
P2) A country has $2n$ cities; n of them are on a line north of the river with x-coordinates $a_1, \ldots, a_n$ and n of them are on a line south of the river with x-coordinates $b_1, \ldots, b_n$. You can assume no two cities in the north have the same coordinates and no two in the south have the same coordinates. We want to make maximum number of bridges between north and south. A bridge is a direct line connecting the i-th city in the north to the i-th city in the south, i.e., a_i to b_i. Design a polynomial time algorithm that outputs the maximum number of bridges we can build such that no two bridges cross each other. For example if $a_1 = 5, a_2 = 2, a_3 = 4$ and $b_1 = 1, b_2 = 4, b_3 = 2$ then, the maximum number of bridges is 1.



P3) Every employee of Apple Inc (except the CEO) has a unique manager (so a manager also has a unique manager etc). Furthermore, you can assume that there is no cycle in the managerial relations, i.e., we cannot have k employees $a_1, \ldots, a_k$ such that a_i is the manager of a_{i-1} and a_1 is the manager of a_k. We want to throw a party and invite a set of employees with the restriction that if we invite an employee we cannot invite his/her manager. Every employee i has a value v_i if it gets invited to the party. Design an algorithm that reads employees' values (including the CEO) and their managerial relations and outputs the sum of the values of the most valuable set of employees to be invited to the party. Your algorithm should run in time polynomial in n.

For example, say 1 is manager of $2, 3$ and 2 is the manger of 4 and 3 is the manager of 5. Furthermore, assume $v_1 = 3, v_2 = 2, v_3 = 2, v_4 = 1, v_5 = 1$. Then you should output 5 corresponding to invite $1, 4, 5$ to the party.

P4) Given a directed graph G such that every edge $e = (u, v)$ sends a packet from u to v with probability $0 < p_e < 1$. Design a polynomial time algorithm that given a pair of vertices s, t finds the path from s to t that has the largest probability of success. You just need to output the probability that a packet from s is successfully sent to t along that path. For example, in the following graph the best path from s to t is s, a, b, t and you should output 0.144.



P5) **Extra Credit:** Given a sequence of positive numbers $x_1, \ldots, x_n$ and an integer k, design a polynomial time algorithm that outputs

$$\sum_{S \in \binom{n}{k}} \prod_{i \in S} x_i,$$

where the sum is over all subsets of size k.