CSE421: Design and Analysis of Algorithms	May 19, 2022
Homework 7	
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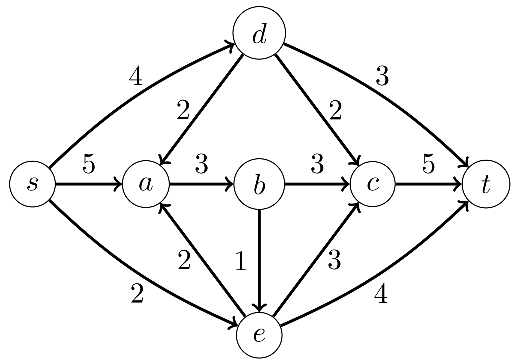
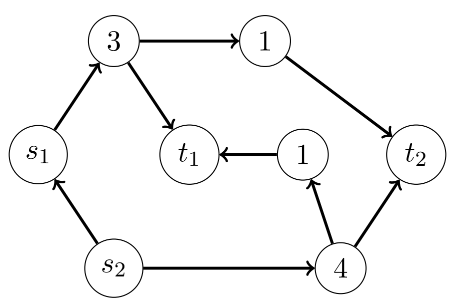
P1) Draw out a maximum s-t flow for the graph below, and the corresponding residual graph G_f. You don't need to show your work. What is the minimum cut that corresponds to this max flow? Write down the capacity of the cut.



P2) Given an directed graph G and k source vertices $s_1, \dots, s_k$ and k destination vertices $t_1, \dots, t_k$ for some $k \geq 1$. You can assume no source vertex is directly connected to a sink vertex. Suppose that each non-source and non-destination vertex v of G has an integer capacity $c_v \geq 0$ which shows the maximum amount of flow that can go through v. You can assume that s_i, t_i's have no capacity. Design an algorithm that runs in time polynomial in $n, \max_v c_v$ and determines the maximum flow that can be sent from all sources to all sinks combined satisfying all capacity constraints of the vertices. For example, in the following picture the maximum amount of flow that can be sent form s_1, s_2 to t_1, t_2 is 7.



P3) In this exercise we give a polynomial time algorithm to find the minimum vertex cover and maximum independent set in a bipartite graph $G = (X, Y, E)$. Following these steps:

a) **Optional:** Construct a flow network H from the given G just as in the maximum matching algorithm that we discuss in class. Simply, draw a graph similar to what we do in class and name different subsets of vertices accordingly.

b) Given a min cut $s-t$ cut (A, B) in H, construct a vertex cover $S \subseteq X \cup Y$ of G such that $|S| = cap(A, B)$

c) Conversely, given a min vertex cover $S \subseteq X \cup Y$ of G, construct a $s-t$ cut (A, B) in H such that $cap(A, B) = |S|$.

d) Write down the algorithm and use the above argument to prove that it correctly finds the min vertex cover of G.

e) **Optional:** Show that for any graph G the complement of the min vertex cover is the maximum independent set. In other words, if S is a minimum vertex cover then $V - S$ is a maximum independent set of G.

P4) Given an $n \times n$ chess board where some cells are removed. Design a polynomial time algorithm to find the maximum number of knights that can be placed on this board such that no two knights attack each other.

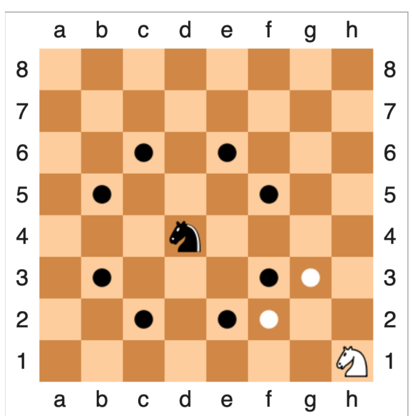
For example in the following 3×3 chessboard the removed cells are marked with X. You can put 4 knights such that no two can attack each other as we did in the right. The location of every knight is marked with a $\bullet$.

X		
		X
	X	

$\Rightarrow$

X		$\bullet$
	$\bullet$	X
$\bullet$	X	$\bullet$

Please see the following image for locations that a knight can attack. In general a knight can attack at most 8 cells if they exist. For example, the white knight can only attack two cells in the following picture.



P5) **Extra Credit:** You are given an $m \times n$ array of real numbers. Suppose that the numbers in each row add up to an integer and the numbers in each column add up to an integer. You want to substitute each number $A[i, j]$ with $\lfloor A[i, j] \rfloor$ or $\lceil A[i, j] \rceil$ such that the sum of the numbers in each row and each column remain invariant. Design a polynomial time algorithm that outputs the integer array.

For example, if the input is the left table you can output the right table. Note the sum of numbers in each row (and each column) of the left table is the same as the sum of the numbers of the same row (resp. the same column) in the right table.

0.4	0.1	1.5
0.6	1.9	0.5

$\Rightarrow$

1	0	1
0	2	1