CSE421: Design and Analysis of Algorithms May 26th, 2022

Homework 8

Shayan Oveis Gharan Due: June 3rd, 2022 at 23:59 PM

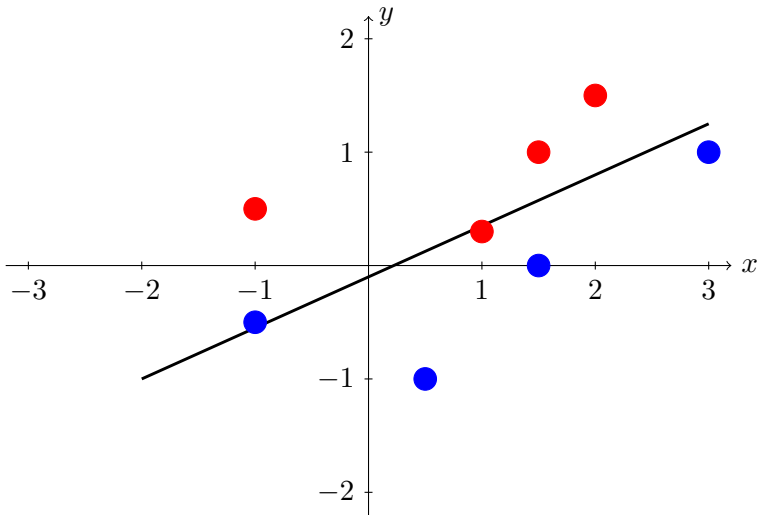
P1) (20 points) Write an Integer Program for the weighted set cover problem. We are given m subsets of the ground set of elements $[n] = \{1, \ldots, n\}$, namely $S_1, S_2, \ldots, S_m$. The cost of choosing S_i is c_i. We want to choose the smallest cost of these sets to cover $[n]$. Note that the cost of choose $S_{i_1}, \ldots, S_{i_k}$ is $c_{i_1} + \cdots + c_{i_k}$. Prove that its optimum solution is equal to the optimum of the weighted set cover. Then, turn it into a LP relaxation for the weighted set cover.

P2) (20 points) Translate the following LP into the standard form. Then write its dual.

$$\begin{aligned} \min \quad & 3x_1 - x_2 \\ \text{s.t.}, \quad & x_1 + x_2 + x_3 = 1 \\ & 2x_1 - x_3 \geq x_2 - 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

No proof is needed.

P3) You want to solve the following linear classification problem using linear programs. You are given n red points $(x_1, y_1), \ldots, (x_n, y_n)$ and n blue points $(x'_1, y'_1), \ldots, (x'_n, y'_n)$. You want to find a half-plane that separates the red from the blue such that the sum the errors is minimized. Namely, for a vector $a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \mathbb{R}^2$ and $b \in \mathbb{R}$ we define the error of each a point (x, y) as $\max\{a_1 x + a_2 y - b, 0\}$ and the error of a blue point (x', y') is $\max\{b - a_1 x' - a_2 y', 0\}$. Write a linear program to find a_1, a_2, b such that the sum of errors over all points is minimized. No proof is needed for this problem.



P4) 4-Color problem is defined as follows: Given a graph $G = (V, E)$, can we color vertices of G with 4 colors such that any two neighbors get distinct colors?

5-Color problem is defined as follows: Given a graph $G = (V, E)$, can we color vertices of G with 5 colors such that any two neighbors get distinct colors?

Prove that 4-Color $\leq_P$ 5-Color.

P5) **Extra Credit:** Prove that the Hamiltonian cycle problem in directed graphs is NP-Complete. You may use the fact that 3SAT is NP-Complete.