1 In-class Exercise

1. Let G be a graph with n vertices and at least n edges. Show that G has a cycle.

2. *Solution:* We prove by contradiction! Suppose G has no cycle. Then,

   Case 1: G is connected. Then since G has no cycles, G is a tree with n vertices. So it must have $n-1$ edges. But we said it has $\geq n$. That is a contradiction.

   Case 2: G is disconnected. Suppose G has ℓ connected components with number of vertices $n_1, n_2, \ldots, n_\ell$ and number of edges $m_1, m_2, \ldots, m_\ell$.

   Claim: For some i we must have $m_i \geq n_i$. **Pf:** For contradiction assume $m_i < n_i$ for all i. Summing up these inequalities we get $m = \sum_i m_i < \sum_i n_i = n$. But that contradicts the assumption that $m \geq n$.

   So assume $m_i \geq n_i$. But then the i-th component is connected and has no cycles. So similar to Case 1 we get a contradiction.