CSE 421 Lecture 8

Question:

Let T be an MST of G, which has unique edge weights. Given a connected subgraph H of G, show that $T \cap G$ is contained in some MST of H.

Fact: If the edge weight is unique, MST is unique due to cut property.

Proof:

Prove it by looking at the Kruskal algorithm on H.

Every step, when the Kruskal look at some edge $e \in T \cap H$.

Case 1: e makes a cycle C (with the tree T_H Kruskal is maintaining)

e is the heaviest edge on the cycle C (because Kruskal picks edges from lowest weight).

So, e violates the cycle property of T on G.

Case 2: e doesn't make a cycle C with T_H.

The algorithm include e to T_H.

This proves the algorithm includes every edge in $T \cap H$.