University of Washington
Department of Computer Science and Engineering
CSE 421, Winter 2024

February 29, 2024

Homework 9, Due Friday, March 8, 11:59 pm, 2024

Turnin instructions: Electronic submission on gradescope using the CSE 421 gradescope site. Submit the assignment as a PDF, with separate pages for different numbered problems. Problems consisting of multiple parts (e.g., 2a, 2b) can be submitted on the same page.

Problem 1 (10 points):

Answer the following questions with "yes", "no", or "unknown, as this would resolve the P vs. NP question." Give a brief explanation of your answer.

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

a) Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

Problem 2 (10 points):

(Kleinberg-Tardos, Page 507, Problem 7). Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define *4-Dimensional Matching* as follows. Given sets W, X, Y, and Z, each of size n, and a collection C of ordered 4-tuples of the form (w_i, x_j, y_k, z_l), do there exist n 4-tuples from C so that no two have an element in common?

Prove that 4-Dimensional Matching is NP-Complete.

Problem 3 (10 points):

(Kleinberg-Tardos, Page 506, Problem 5). Consider a set $A = \{a_1, \ldots, a_n\}$ and a collection $B_1, B_2, \ldots, B_m$ of subsets of A (i.e., $B_i \subseteq A$ for each i).

We say that a set$H \subseteq A$ is a *hitting* set for the collection $B_1, B_2, \ldots, B_m$ if H contains at least one element from each B_i – that is, if $H \cap B_i$ is not empty for each i (so H "hits" all the sets B_i).

We now define the *Hitting Set Problem* as follows. We are given a set $A = \{a_1, \ldots, a_n\}$, a collection $B_1, B_2, \ldots, B_m$ of subsets of A, and a number k. We are asked: Is there a hitting set $H \subseteq A$ for $B_1, B_2, \ldots, B_m$ so that the size of H is at most k?

Prove that Hitting Set is NP-complete.

Problem 4 (10 points):

(Kleinberg-Tardos, Page 506, Problem 6). Consider an instance of the Satisfiability Problem, specified by clauses $C_1, \ldots, C_k$ over a set of Boolean variables $x_1, \ldots, x_n$. We say that the instance is *monotone* if each term in each clause consists of a nonnegated variable; that is, each term is equal to x_i, for some i, rather than $\overline{x_i}$. Monotone instances of Satisfiability are very easy to solve: They are always satisfiable, by setting each variable equal to 1.

For example, suppose we have the three clauses

$$(x_1 \vee x_2), (x_1 \vee x_3), (x_2 \vee x_3).$$

This is monotone, and indeed the assignment that sets all three variables to 1 satisfies all the clauses. But we can observe that this is not the only satisfying assignment; we conld also have set x_1 and x_2 to 1, and x_3 to 0. Indeed, for any monotone instance, it is natural to ask how few variables we need to set to 1 in order to satisfy it.

Given a monotone instance of Satisfiability, together with a number k, the problem of Monotone Satisfiability with Few True Variables asks: Is there a satisfying assignment for the instance in which at most k variables are set to 1? Prove this problem is NP-complete.

Problem 5 (10 Points):

(Kleinberg-Tardos, Page 513, Problem 17). You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The *Zero-Weight-Cycle Problem* is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that the Zero-Weight-Cycle problem is NP-Complete. (Hint: Hamiltonian PATH)