Lecture04

CSE 421

Introduction to Algorithms

Winter 2024

Lecture 4

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Announcements

- Reading
  - Start on Chapter 4
- Homework due tonight, new homework available
- Class Friday???
- No class next Monday (MLK)

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Graph Theory

- $G = (V, E)$
  - V: vertices, $|V| = n$
  - E: edges, $|E| = m$
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops
- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Neighborhood
  - N(v)
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

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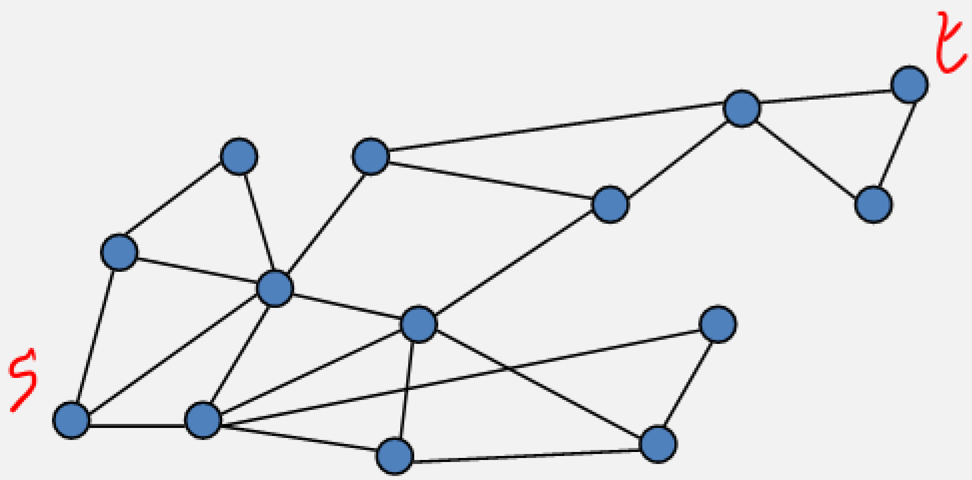
Last Lecture

- Bipartite Graphs : two-colorable graphs
- Breadth First Search algorithm for testing two-colorability
  - Two-colorable iff no odd length cycle
  - BFS has cross edge iff graph has odd cycle

Graph Search

- Data structure for next vertex to visit determines search order



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Graph search

Breadth First Search

```
S = {s}
while S is not empty
    u = Dequeue(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Enqueue(S, v)
```

Depth First Search

```
S = {s}
while S is not empty
    u = Pop(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Push(S, v)
```

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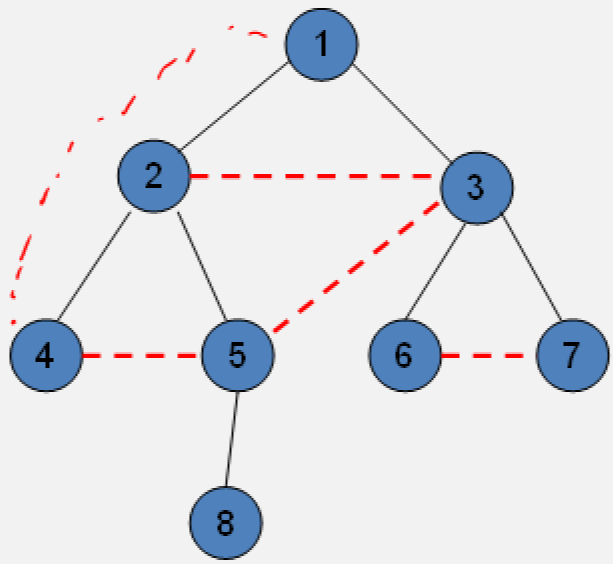
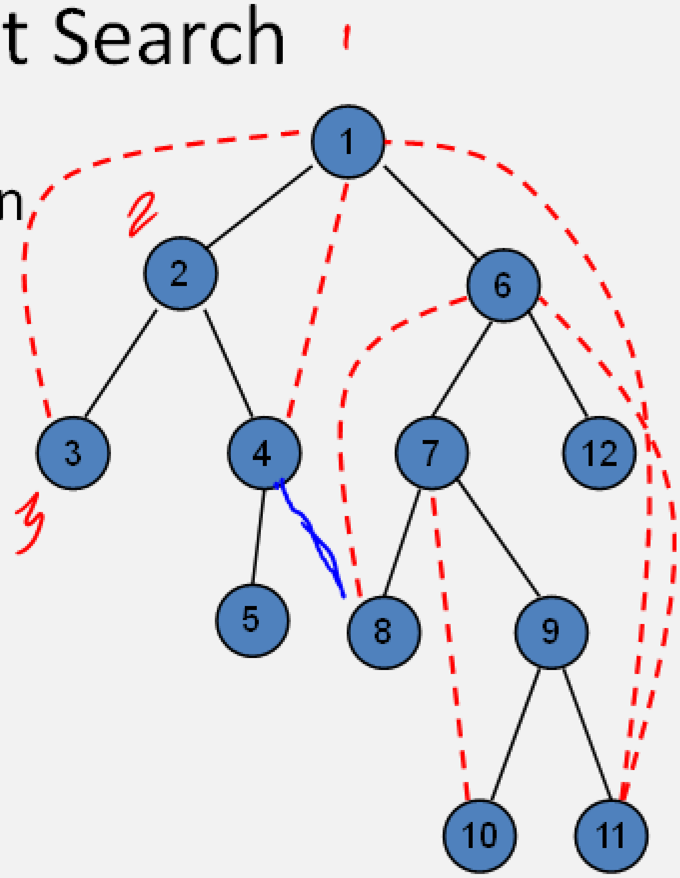
Breadth First Search

- All edges go between vertices on the same layer or adjacent layers



Depth First Search

- Each edge goes between vertices on the same branch
- No cross edges



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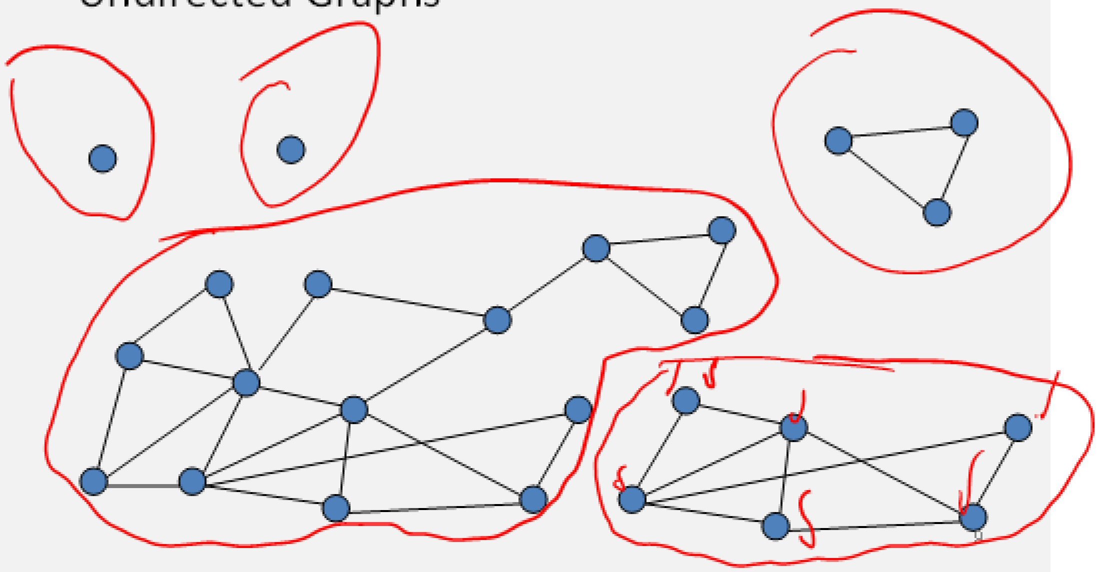
$O(n+m)$

Connected Components

- Undirected Graphs



Computing Connected Components in O(n+m) time

- A search algorithm from a vertex v can find all vertices in v's component
- While there is an unvisited vertex v, search from v to find a new component

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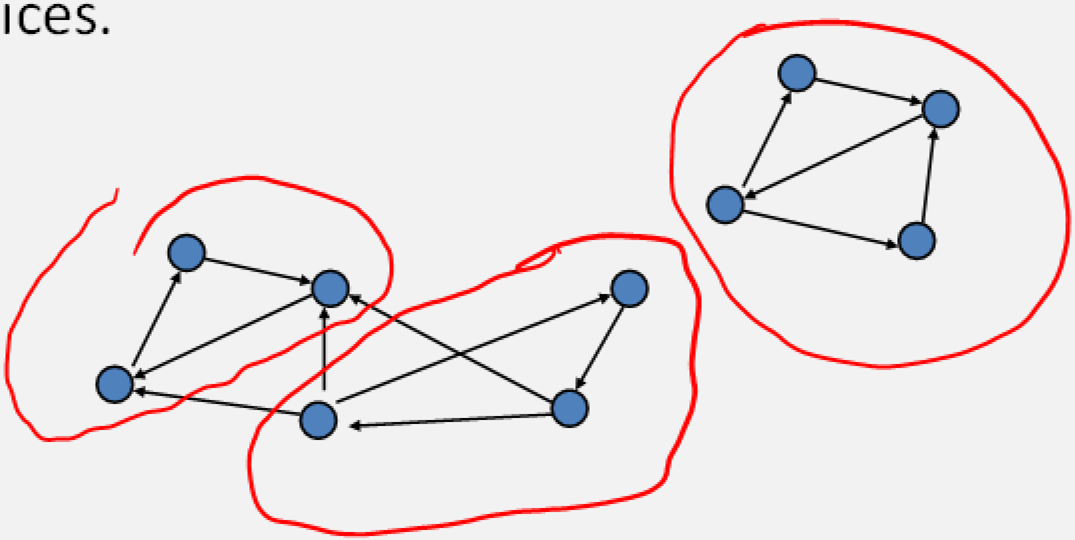
Directed Graphs

- A Strongly Connected Component is a subset of the vertices with paths between every pair of vertices.



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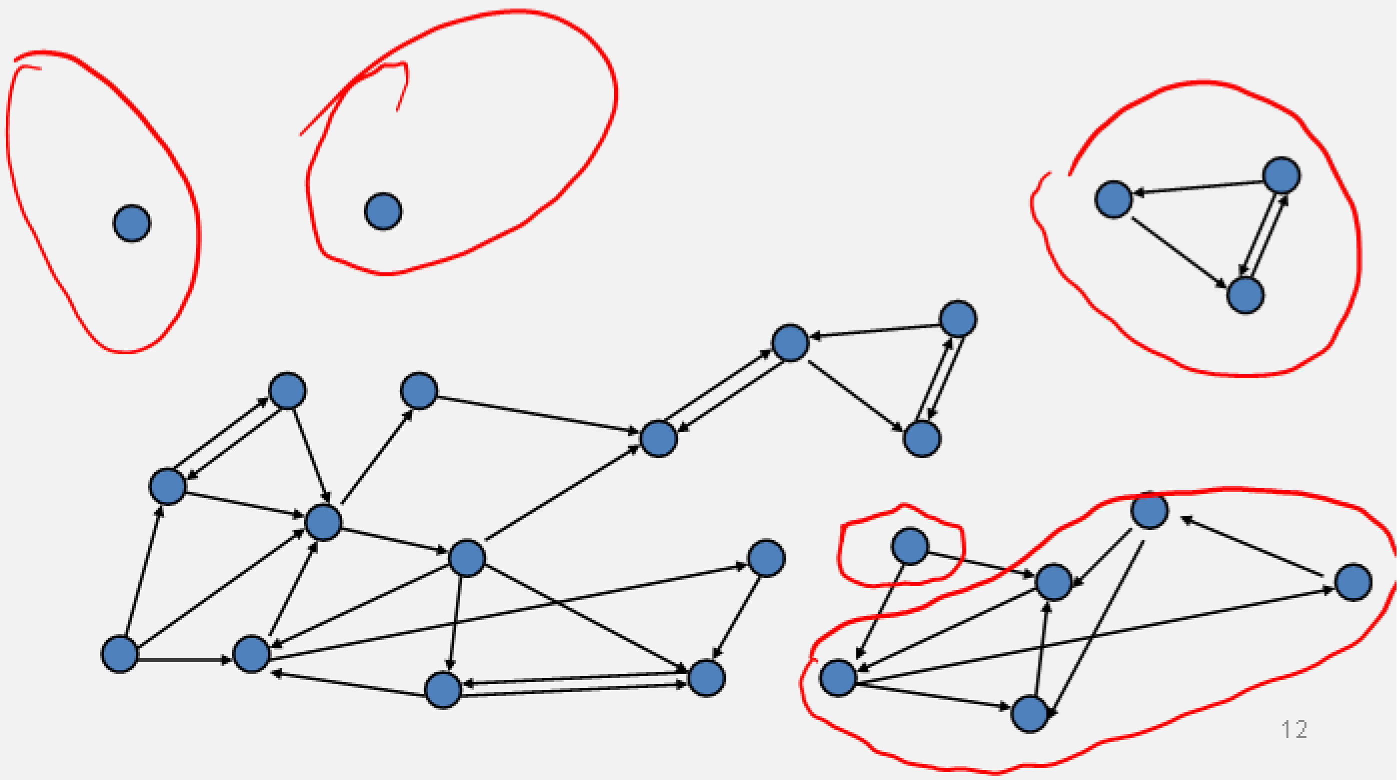
Identify the Strongly Connected Components



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Strongly connected components can be found in O(n+m) time

- But it's tricky!
- Simpler problem: given a vertex v, compute the vertices in v's scc in O(n+m) time

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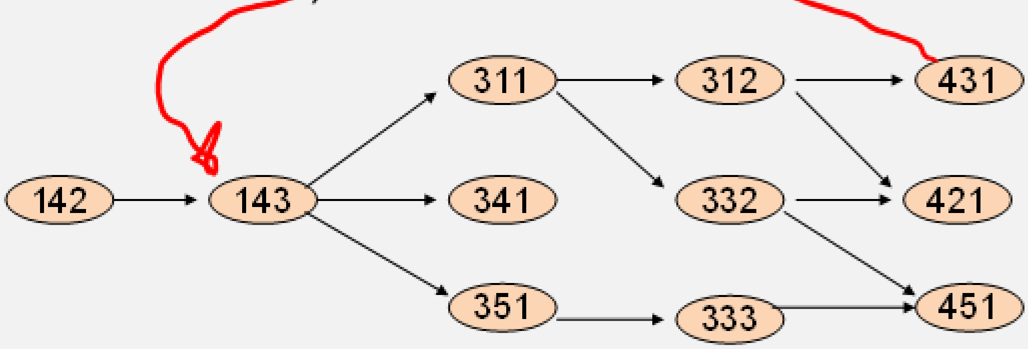
Topological Sort

- Given a set of tasks with precedence constraints, find a linear order of the tasks



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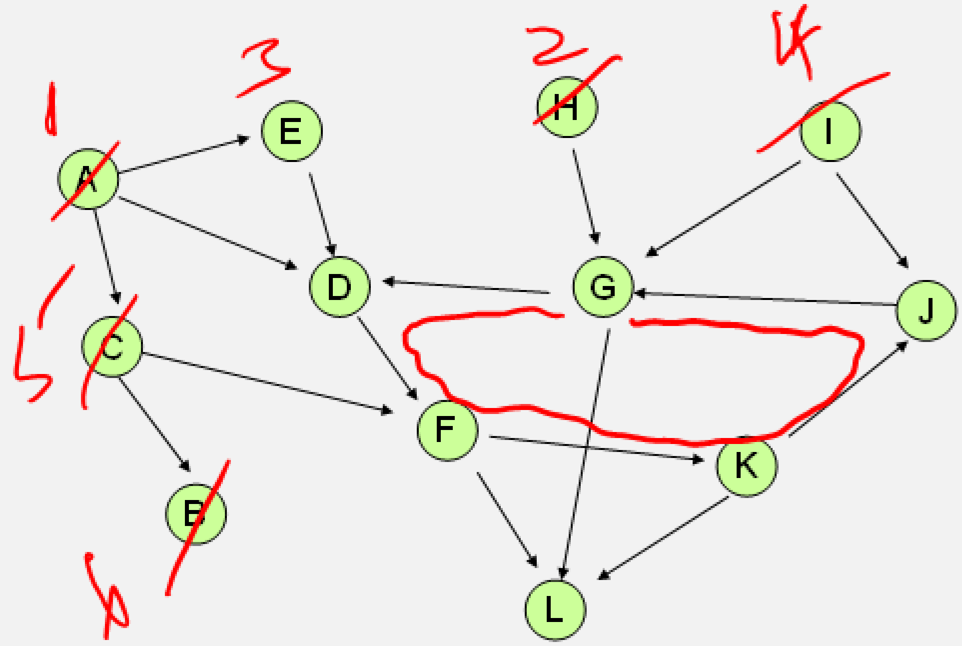
Find a topological order for the following graph



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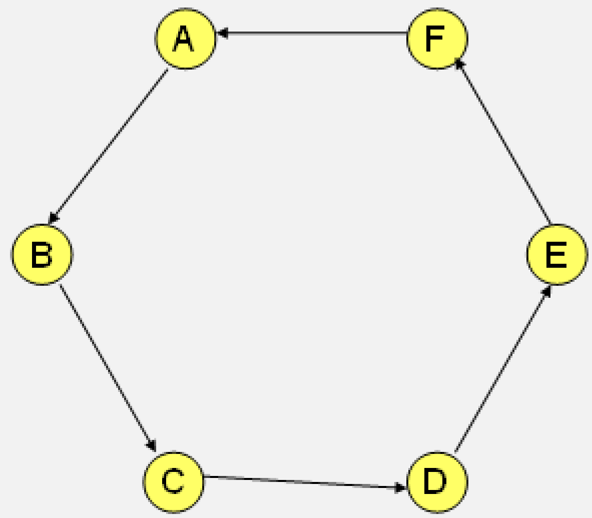
If a graph has a cycle, there is no topological sort

- Consider the first vertex on the cycle in the topological sort
- It must have an incoming edge



Definition: A graph is Acyclic if it has no cycles

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Lemma: If a ~~(finite)~~ graph is acyclic, it has a vertex with in-degree 0

- Proof:
  - Pick a vertex v_1, if it has in-degree 0 then done
  - If not, let (v_2, v_1) be an edge, if v_2 has in-degree 0 then done
  - If not, let (v_3, v_2) be an edge . . .
  - If this process continues for more than n steps, we have a repeated vertex, so we have a cycle

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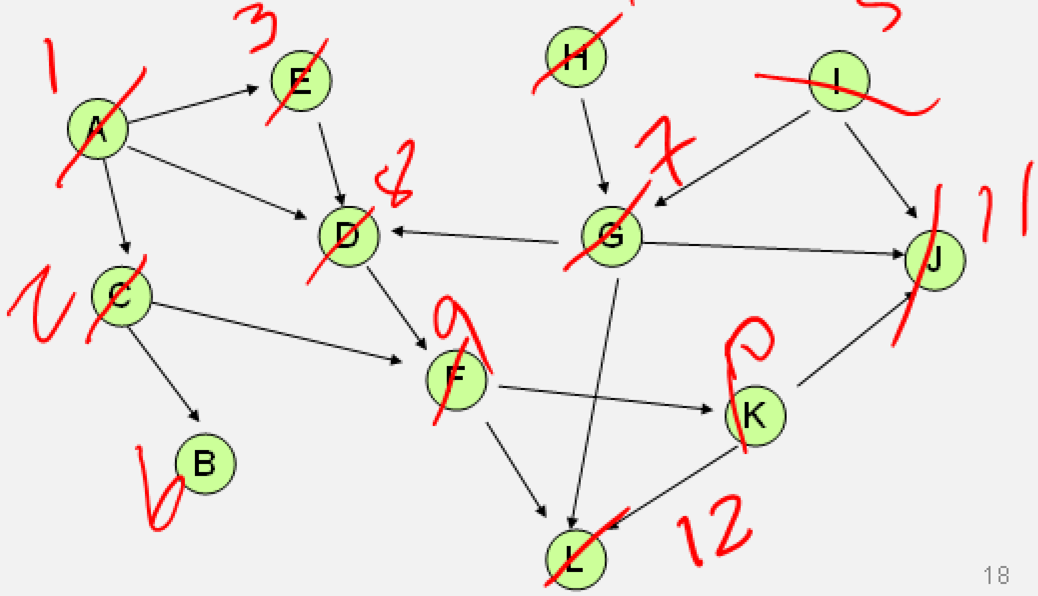
Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

Delete the vertex v and all out going edges



Details for O(n+m) implementation

- Maintain a list of vertices of in-degree 0
- Each vertex keeps track of its in-degree
- Update in-degrees and list when edges are removed
- m edge removals at O(1) cost each