CSE 421 Section 10

P, NP, Reductions

Administrivia

Announcements & Reminders

- Last Homework!
- Due Friday!
- Final Exam on **Monday, March 11, 2:30-4:20 PM!**
- Sample exams on course website!

NP-Completeness

Precedence graph construction

- A problem X is NP-complete if
- 1. X is in NP
- 2. For every Y in NP, Y $<_p$ X

- X is a "hardest" problem in NP

- If X is NP-Complete, Z is in NP and X $<_p$ Z. Then Z is NP-Complete

Important NPC

- The Circuit Satisfiability Problem
- 3 SAT
- Independent Set
- Vertex Cover
- Integer Programming
- Hamiltonian Circuit
- Graph Coloring
- Subset sum

Reductions

Efficient Recruiting Problem

- Suppose you're helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who's skilled at each of the n sports covered by the camp (baseball, volleyball, and so on). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is: For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports?

Vertex Cover!

- Consider the definition of vertex cover:
- A set of vertices that includes at least one endpoint of every edge of the graph

- Each edge is a sport
- Each vertex is a coach
- The sport represented by the edge has its 2 vertices as only 2 candidates.

- Now we show vertex cover $<_p$ Efficient Recruiting Problem

Strong Independent Set

The following is a version of the Independent Set Problem. You are given a graph G=(V,E) and an integer k. For this problem, we will call a set I in V strongly independent if, for any two nodes v, u in l, the edge (v, u) does not belong to E, and there is also no path of two edges from u to v, that is, there is no node w such that both (u, w) in E and (w, v) in E. The Strongly Independent Set Problem is to decide whether G has a strongly independent set of size at least k.

Prove that the Strongly Independent Set Problem is NP-complete.

Independent Set!

- One very simple intuition is adding a vertex between each edges (u,v) to (u,w),(w,v), so distance 1 becomes distance 2 now.
- But how can we avoid choosing the added in our solution?

- We can add edges to all pair of new vertex. After doing this, the distance from a new vertex to any vertex is less than 2! So if we choose one from the new vertices, we can’t choose any other vertex.
- At the same time, the pair of original vertices that have distance more than 2 will still be more than 2.

- Now we show independent set $<_p$ strong independent set

That’s All, Folks!

Thanks for coming to section this week!
Any questions?