# CSE 431
# Spring Quarter 2001
# Assignment 6
# Due Friday, May 18

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) It is well known that the Hamiltonian Path problem is NP-complete (cf. 262-268 of Sipser). The Hamiltonian Path problem is defined by:

   | | |
   |---|---|
   | Input: | An undirected graph $G = (V, E)$ and vertices $s, t \in V$. |
   | Property: | There is a path in $G$ from $s$ to $t$ that visits every vertex of $G$ exactly once. |

   Show that the problem of Bounded Degree Spanning Tree is also NP-complete. Bounded Degree Spanning Tree problem is defined by:

   | | |
   |---|---|
   | Input: | A connected undirected graph $G = (V, E)$ and number $k$. |
   | Property: | There is a connected subgraph $T$ of $G$ such that $T$ contains all the vertices of $G$, contains no cycles, and each vertex of $T$ has degree $\leq k$. |

   Part of your proof should be the construction of a polynomial time reduction of Hamiltonian Path to Bounded Degree Spanning Tree.

2. (10 points) It is well known that the Subset Sum problem is NP-complete (cf. 268-270 of Sipser). The Subset Sum problem is defined by.

   | | |
   |---|---|
   | Input: | A sequence of numbers $a_1, a_2, \ldots, a_n$ and a number $b$ all written in binary. |
   | Property: | There is a set $S \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in S} a_i = b$. |

   Show that the Equal Partition problem is NP-complete where it is defined by:

   | | |
   |---|---|
   | Input: | A sequence of numbers $c_1, c_2, \ldots, c_n$ all written in binary. |
   | Property: | There is a set $S \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in S} c_i = \sum_{j \notin S} c_j$. |

   Part of your proof should be the construction of a polynomial time reduction of Subset Sum to Equal Partition.