CSE 431
Spring Quarter 2002
Assignment 1
Due Friday, April 12, 2002

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) This problem is designed to give you practice in coding Turing machines. Use a state diagram to define a Turing machine that increments a "counter" on binary strings. By counting I mean in the enumeration ordering of binary strings $\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots$. Suppose that a Turing machine tape has $\#w$ (followed by infinitely many blanks) written on the tape where w is a binary string and the read head is on the first symbol after the $\#$. The Turing machine halts when after writing $\#w'$ on the tape were w' is the next string after w in the enumeration ordering of binary strings. For example, if the machine starts with tape contents $\#\underline{1}011 \sqcup \cdots$ then it halts with tape contents $\#\underline{1}100 \sqcup \cdots$ (the underline indicates the head position). In another example, if the machine starts with tape contents $\#\underline{1}111 \sqcup \cdots$ then it halts with tape contents $\#\underline{0}0000 \sqcup \cdots$. Use an example to demonstrate how your machine operates.

2. (10 points) This is another problem designed to give you practice in coding Turing machines. In this case it will be a two-tape Turing machine. Define x is a substring of y if there are strings u and v such that $y = uxv$. Note that the empty string is always a substring of any string and that a string is always a substring of itself. Use a state diagram to define a two-tape Turing machine that decides the language

   $$L = \{x\#y : x, y \in \{0,1\}^* \text{ and } x \text{ is a substring of } y\}.$$

   Give a state diagram for your Turing machine. The first thing your machine should do is copy x to the second tape. The alphabet of L is $\{0, 1, \#\}$. Use an example to demonstrate how your machine operates.

3. (10 points) An alternative definition of the storage for a Turing machine is a two-way infinite tape. That is, the tape does not have a beginning or an end. For such a machine we assume that the input is written some place on the tape, with blanks to the left and to the right. The head starts on the first symbol of the input. Given a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$ with a two-way infinite tape, design a Turing machine M' with a one-way infinite tape such that $L(M') = L(M)$. Your design should give the specific components of $M' = (Q', \Sigma, \Gamma', \delta', q_0', q_a', q_r')$.