CSE 431
Spring Quarter 2002
Assignment 7
Due Friday, May 31

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) The following algorithm solves the Subset Sum problem in pseudo polynomial time. With input `a[1],...,a[n],b` do the following. Let `X` be an array of Boolean values with range `0` to `b`.

```
for j = 1 to b do X[j] = 0;
X[0] = 1;
for i = 1 to n do
  for j = b to 0 by -1 do
    if X[j] == 1 and j + a[i] <= b then
      X[j + a[i]] = 1;
return X[b]
```

   Unfortunately, this algorithm only determines whether or not there is a solution. Use this algorithm as a basis of an algorithm to solve the problem of finding a solution to the Subset Sum problem if there is one. Your algorithm should run in time $O(bn)$.

2. (10 points) In this problem you will learn about *resolution*, a technique for determining satisfiability of CNF formulas. For resolution we represent a clause as a set of literals. For example, the clause $x \vee \neg y \vee z$ is represented as the set $\{x, \neg y, z\}$. A CNF formula is then represented as a set of such sets. For example the formula

$$(x \vee \neg y \vee z) \wedge (\neg x \vee y) \wedge (x \vee y \vee \neg z)$$

   is represented as the set of sets

$$\{\{x, \neg y, z\}, \{\neg x, y\}, \{x, y, \neg z\}\}.$$

   From now on we represent CNF formulas in this way. A resolution step is the following operation on a CNF. If $C \cup \{x\}$ and $D \cup \{\neg x\}$ are two distinct clauses in F, then add $C \cup D$ to F provided the new clause does not contain any complementary literals. The clause $C \cup D$ is called the resolvent of $C \cup \{x\}$ and $D \cup \{\neg x\}$. For example, the resolvent of the first two clauses in the CNF above is $\{\neg y, z, y\}$. This clause contains complementary literals so it is not added to the set of clauses. In another example, the resolvent of the last two clauses is $\{y, \neg z\}$ which is added to the set of clauses to yield

$$\{\{x, \neg y, z\}, \{\neg x, y\}, \{x, y, \neg z\}, \{y, \neg z\}\}.$$

Note that the resolution process eventually terminates when every possible resolvent is a member of F. There are at most 2^{2n} clauses with n variables. Resolution has the property that the original formula F is satisfiable if and only if after the resolution process the empty set is not a member of F. Thus, resolution is essentially an algorithm for deciding CNF satisfiablilty.

(a) Use resolution to show that 2-CNFSAT is decidable in polynomial time.

(b) Construct a polynomial time reduction from 2-COLOR to 2-CNFSAT to demonstrate that 2-COLOR is also decidable in polynomial time. Recall, that 2-COLOR is the problem of given a graph determining if its vertices can be colored in two colors so that no two adjacent vertices have the same color.