CSE 431
Spring Quarter 2004
Assignment 3
Due Friday, April 23

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem you will explore the *Turing machine enumerator* (pages 140-141 of Sipser). A Turing machine enumerator M is a two tape Turing machine where both tapes are initially empty. The first tape is a work tape and the second is a write-only output tape. The enumerator M runs forever and in process outputs a string $w_1\#w_2\#\cdots$ on its output tape, where each $w_i \in \Sigma^*$ and $\# \notin \Sigma$. The set $\{w_1, w_2, w_3, \ldots\}$ is the language enumerated by the machine. In the book it is shown that a language is enumerated by a Turing machine enumerator if and only if it is Turing recognizable.

   (a) Suppose a Turing machine enumerator M outputs $w_1\#w_2\#w_3\#\cdots$ with the property that for all $i \geq 1$, $|w_i| < |w_{i+1}|$. Argue that the language enumerated by M is decidable.

   (b) Use the result in (a) and the equivalence of Turing enumeration and Turing recognition to show that every infinite Turing recognizable language has an infinite decidable subset.

2. (10 points) In this problem you will get practice in doing a reduction argument to show undecidability. Consider the language
$$I_{TM} = \{\langle M \rangle : \text{ both } L(M) \text{ and its complement are infinite}\}.$$
Show that I_{TM} is undecidable by a reduction from A_{TM}.

3. (15 points) Thue systems were invented in 1914. A Thue system consists of a finite set of rules T of the form
$$(u_1, v_1), (u_1, v_2), \ldots, (u_n, v_n)$$
where u_i, v_i are strings from a finite alphabet Σ. One string can be derived in one step from another via T using the following definition
$$ux \Rightarrow xv \text{ if for some } i, u = u_i \text{ and } v = v_i.$$
For example, suppose the rules in T are
$$(0,0), (1,1), (\#,\#), (c1, 0c), (c0, d1), (c\#, d0\#), (0d, d0), (1d, d1), (\#d, \#c)$$
then we could have the multiple step derivation
$$\#c\# \Rightarrow c\#\# \Rightarrow \#d0\# \Rightarrow 0\#\#c \Rightarrow \#\#c0 \Rightarrow \#c0\# \Rightarrow$$
$$c0\#\# \Rightarrow \#\#d1 \Rightarrow \#d1\# \Rightarrow 1\#\#c \Rightarrow \#\#c1 \Rightarrow \#c1\#$$

(a) Continue the derivation above for 10 more steps. Describe in words what the Thue system T is doing.

(b) Show that the problem of determining if given Thue system T and start string x, x derives the empty string in T, is undecidable. Hint: Do a reduction from the acceptance problem for Turing machines. That is, given a Turing M and input w show how to construct a Thue system T and start string x with the property that M accepts w if and only if x derives the empty string in T. There will be some resemblence to the construction of a general grammar equivalent to a given Turing machine.