CSE 431 Spring 2009
Assignment #2

Due: Friday, April 17, 2009

Reading assignment: Read Chapter 4 of Sipser's text.

Problems:

1. Show that both the decidable languages and the Turing recognizable languages are closed under the concatenation, intersection, and star operations.

2. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)

3. Use the result of question 2 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

4. Let $INFINITE_{PDA} = \{\langle M \rangle \mid M$ is a PDA and $L(M)$ is an infinite language$\}$. Show that $INFINITE_{PDA}$ is decidable.

5. (a) Show that the set of complex numbers,

   $QUADRATIC\text{-}ROOT = \{x \in \mathbb{C} \mid$ there are integers $a \neq 0$, b, and c such that $ax^2+bx+c = 0\}$

   is countable.

   (b) Show that the set of complex numbers

   $ALGEBRAIC\text{-}NUMBERS = \{x \in \mathbb{C} \mid$ there is a non-zero polynomial p with integer coefficients such that $p(x) = 0\}$

   is countable.

6. (Bonus) Let C be a language. Prove that C is Turing-recognizable iff there is a decidable language D such that $C = \{x \mid \exists y$ such that $\langle x, y \rangle \in D\}$.