CSE 431 Spring 2009
Assignment #4

Due: Friday, May 1, 2009

Reading assignment: Finish reading Chapter 5 of Sipser's text and skim sections 6.3 and 6.4 of the text.

Problems:

1. Show that there is a undecidable language contained in 1^*.
2. Let $S = \{\langle M \rangle \mid |L(M)| \text{ is even}\}$. Prove that neither S nor $\overline{S}$ is Turing-recognizable.
3. Which of the following problems are decidable? Justify each answer:
   (a) Given Turing machines M and N, is $L(N)$ the complement of $L(M)$?
   (b) Given a Turing machine M, does M only accept binary encodings of prime numbers?
   (c) Given a Turing machine M, integers a and b, and input x, does M run for more than $a|x|^2 + b$ steps on input x?
   (d) Given a program P written in Java, or C, or (insert your favorite programming language) that does not read any input but is executed with no bound on the size of integers, does P ever attempt to index an array outside its allocated array bounds.
4. Sipser's text: 1st edition problem 5.19; 2nd edition problem 5.21.
5. (Extra Credit) Show that the following problem is undecidable: Given a Turing machine M and integers a and b, does there exist an input x on which M runs for more than $a|x|^2 + b$ steps on input x?
6. (Extra Credit) Rice's Theorem shows that for every 'non-trivial' property $\mathcal{P}$ of languages,

   $$\mathcal{P}_{TM} = \{\langle M \rangle \mid L(M) \text{ has property } \mathcal{P}\}$$

   is undecidable where by '$\mathcal{P}$ is non-trivial' we mean that $\mathcal{P}$ contains some but not all Turing-recognizable languages. Some of these $\mathcal{P}_{TM}$ are not only undecidable, they are also not Turing-recognizable:

   Show that if there is some *infinite* Turing-recognizable language L that has property $\mathcal{P}$ but none of the finite subsets of L have property $\mathcal{P}$ then $\mathcal{P}_{TM}$ is not Turing recognizable.