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CSE 431
Introduction to Theory of Computation
Homework #5
Due: Friday, May 14, 2010

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Suggestion: Work, but do NOT hand in Exercises 7.1, 7.3, 7.5.

Homework Assignment:

1. For any language $L \in \Sigma^*$, let $ be a symbol not in Σ and let

$$\mathrm{PAD}(L) = \{x\$^k \mid x \in L \text{ and } k = 2^{|x|} - |x|\}.$$

   Suppose $L \in \mathrm{MTM\text{-}TIME}(2^n)$. Prove that $\mathrm{PAD}(L) \in \mathrm{MTM\text{-}TIME}(n)$. Here, "MTM-TIME" means time on a multi-tape TM. (Hint: Be clear about what "n" means.)

2. 7.6. Omit closure under union.

3. 7.7. Omit closure under union.

4. 7.9.

5. Suppose $f : \Sigma^* \to \Gamma^*$ is computable in time $O(n^2)$ and $g : \Gamma^* \to \Delta^*$ is computable in time $O(n^3)$. The function g composed with f, denoted $g \circ f$, is defined to be the function having $(g \circ f)(x) = g(f(x))$ for all $x \in \Sigma^*$. Note that $(g \circ f) : \Sigma^* \to \Delta^*$.

   (a) Show that $g \circ f$ is a polynomial time computable function.

   (b) What is the time bound for computing that function? (Hint: it is neither $O(n^2)$ nor $O(n^3)$. Be clear about what "n" means.)

   (c) Prove that $\leq_P$ is transitive.