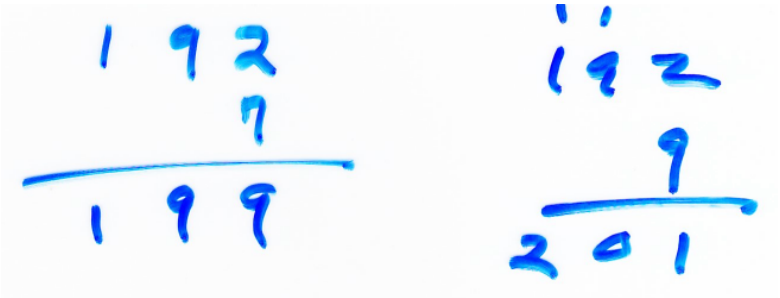
Lecture 2

Algorithms

"An *algorithm* is a finite, precise set of instructions for performing a computation"



"The Division Algorithm": $\forall$ a $\in \mathbb{Z}$, d $\in \mathbb{Z}^+$,

$\exists$ unique q, r such that $0 \leq r < d$ and $a = qr+d$

Deal with finite set of discrete objects

Finite list of instructions

Each

Simple

unambiguous

from a finite set of possibilities

"clearly" solvable in finite

time by "simple" agent

Overall process will finish in

a finite amount of time.

Examples

$x \div 1$

$x \div y$

$x * y$

String concatenation

__Defn__ $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

Q: finite __state set__

Σ: finite __input alphabet__ set ; $\sqcup \notin \Sigma$ ("blank")

Γ: finite __tape alphabet__ . $\Sigma \cup \{\sqcup\} \subseteq \Gamma$.

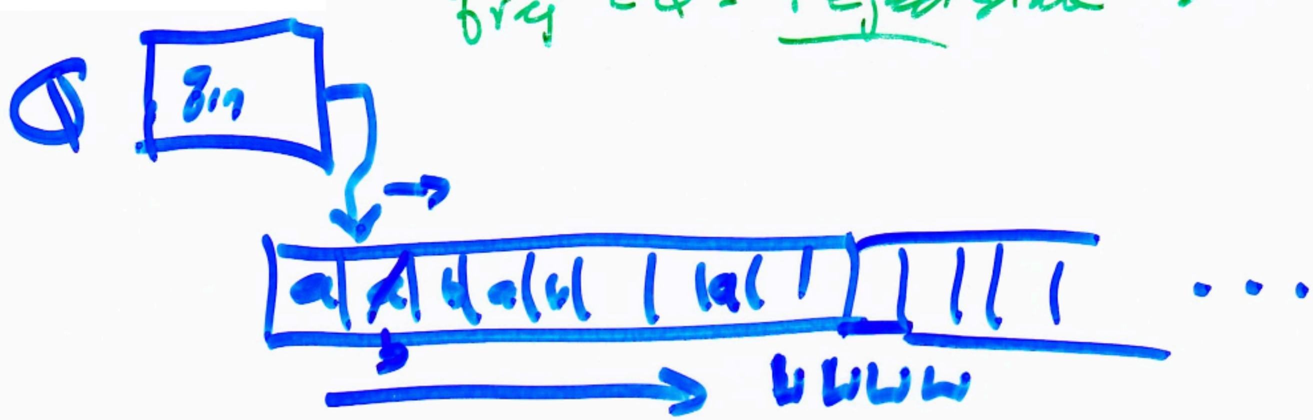
δ: $Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ __transition function__

$q_0 \in Q$: __start state__

$q_{acc} \in Q$: __accept state__

$q_{rej} \in Q$: __reject state__

) $\neq$



Example

$$L = \{ w\#w \mid w \in \{0,1\}^* \}$$

1. check that there's a single #
2. read, remember & cross off leftmost uncrossed letter
3. scan to # & compare next uncrossed letter
4. If OK, cross it off
5. repeat