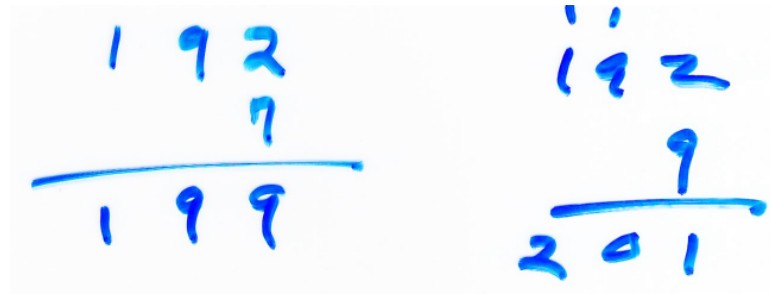


Lecture 2

Algorithms

“An *algorithm* is a finite, precise set of instructions for performing a computation”



Two handwritten division problems in blue ink. The first problem shows 192 divided by 7, with a horizontal line above the 9 and 9, and the result 199 written below. The second problem shows 192 divided by 9, with a horizontal line above the 9, and the result 201 written below.

“The Division Algorithm”: $\forall a \in \mathbb{Z}, d \in \mathbb{Z}^+,$
 \exists unique q, r such that $0 \leq r < d$ and $a = qr + d$

Deal with finite set of discrete objects

Finite list of instructions

Each

Simple

unambiguous

from a finite set of possibilities,

"clearly" solvable in finite

time by "simple" agent

Overall process will finish in
a finite amount of time.

Examples

$x \div 1$

$x \div y$

$x \neq y$

Strong concentration

Defn $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

Q : finite state set

Σ : finite input alphabet set ; $\sqcup \notin \Sigma$ ↖ "blank"

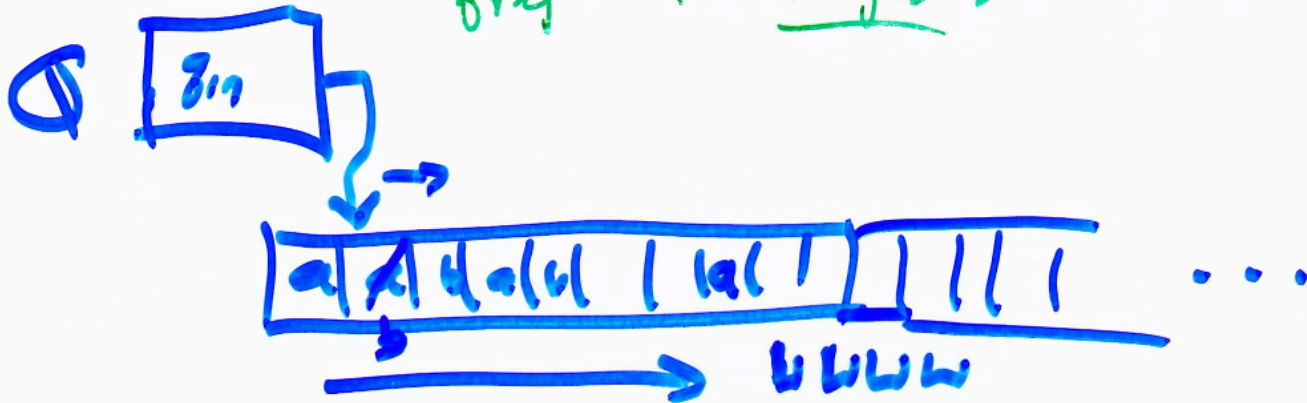
Γ : finite tape alphabet . $\Sigma \cup \{\sqcup\} \subseteq \Gamma$.

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ transition function

$q_0 \in Q$: start state

$q_{acc} \in Q$: accept state) \neq

$q_{rej} \in Q$: reject state)



Example

$$L = \{ w \# w \mid w \in \{0,1\}^* \}$$

1. check that there's a single #
2. read, remember & cross off
left most ^{uncrossed} letter
3. scan to # & compare next ^{uncrossed} letter
4. If OK, cross it off
5. repeat