Lecture 4

Announcements

Late policy

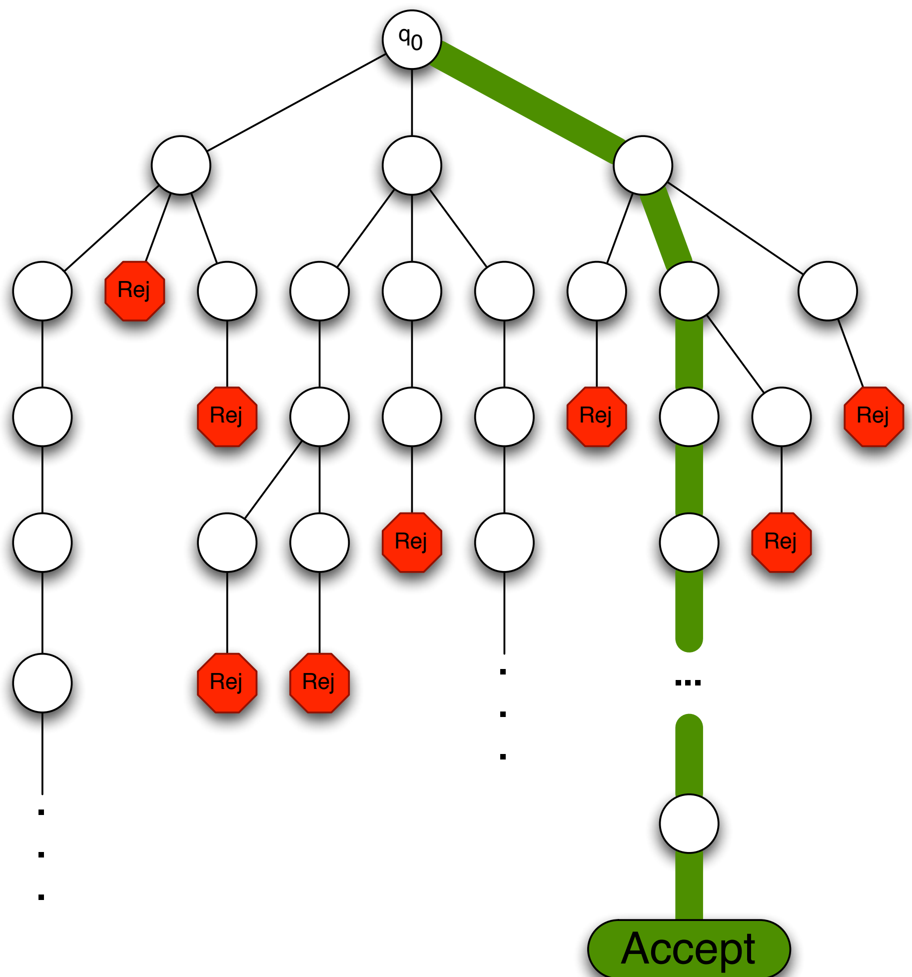
eTurnin

Office hours M 2:30, W 12:30, Th 5:00

Midterm Fri 5/7, probably

Nondeterministic Turing Machines

$$\delta: Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L,R\})$$

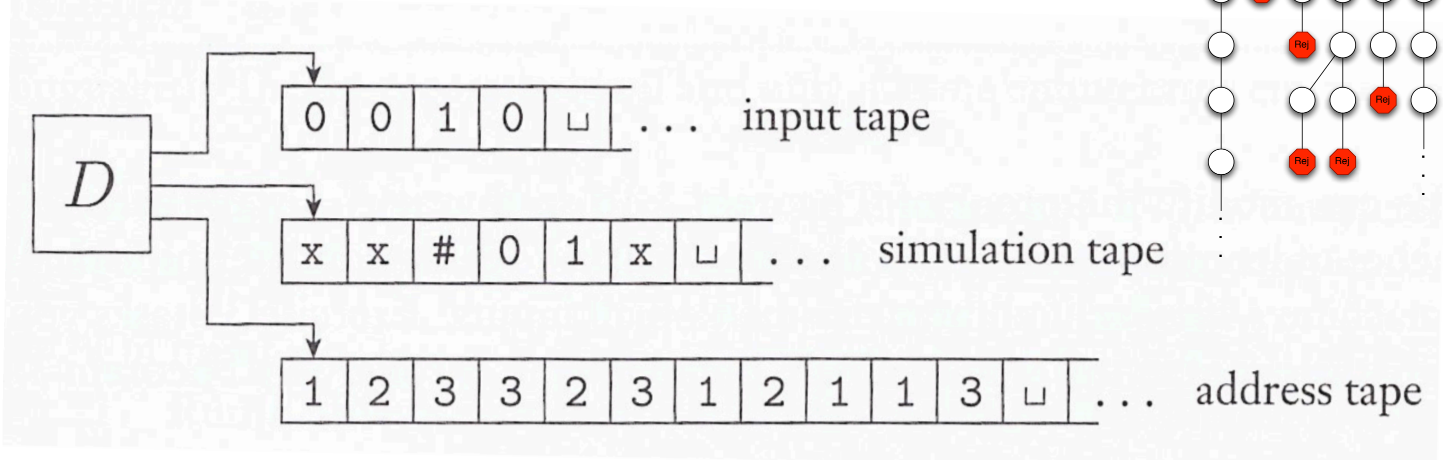


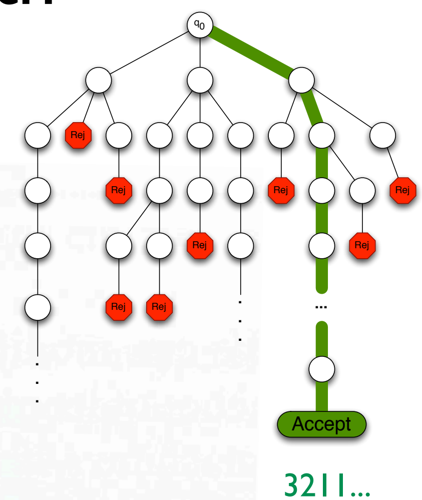
Accept if *any* path leads to q_{accept}; reject otherwise, (i.e., *all* halting paths lead to q_{reject})

Simulating an NTM

Key issue: avoid getting lost on ∞ path

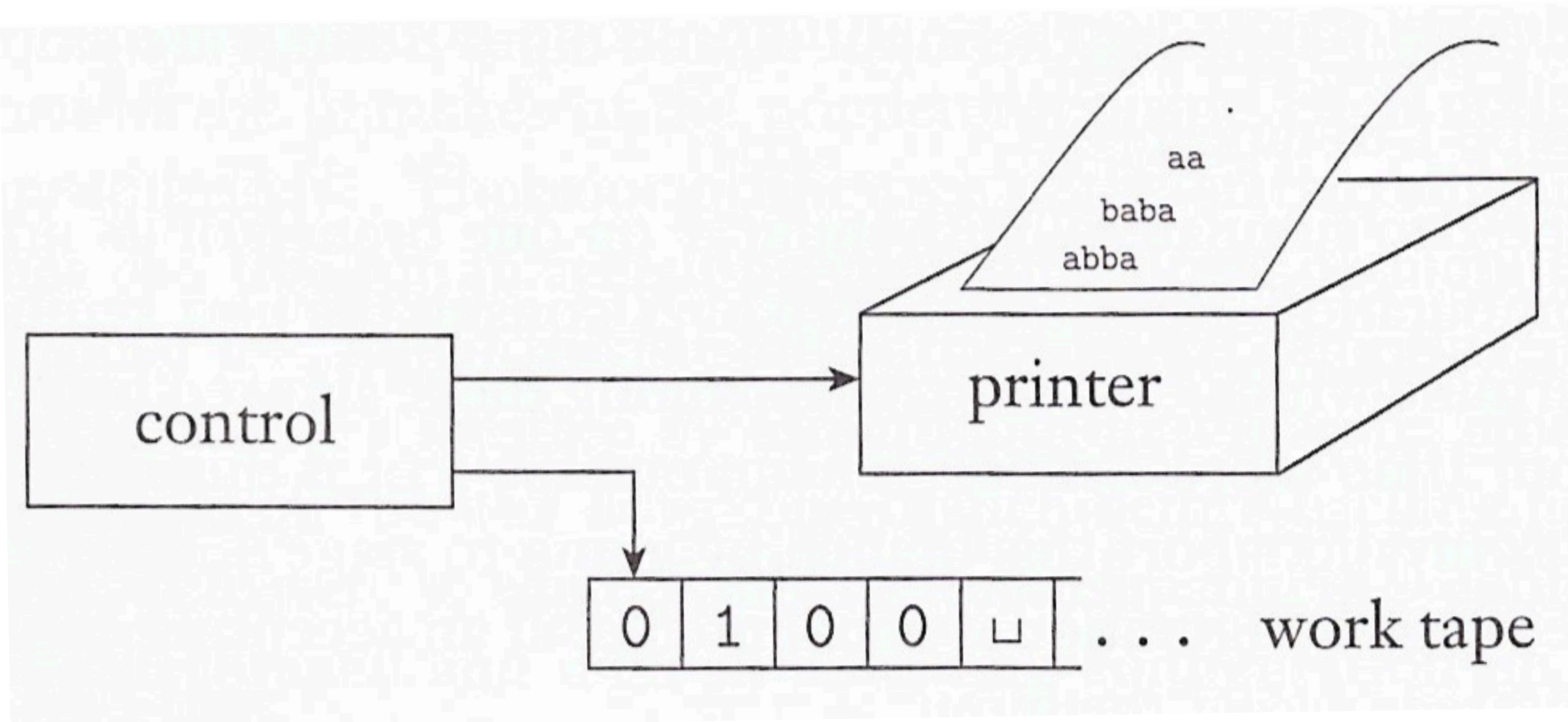
Key Idea: *breadth*-first search





tree arity ≤ |Q| x | Γ| x |{L,R}| (3 in example)

A TM "Enumerator"



L Turing recognizable iff a TM enumerates it

(⇐): Run enumerator, compare each "output" to input; accept if they match (reject by not halting if input never appears)

(⇒): The "obvious" idea: enumerate Σ^*, run the recognizer on each, output those that are accepted.

[Oops, doesn't work...]

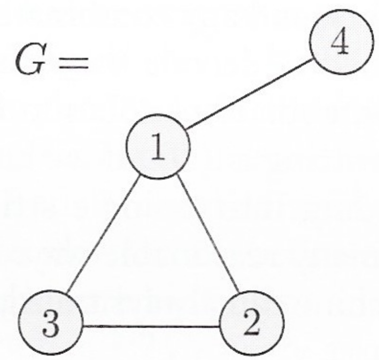
L Turing recognizable iff a TM enumerates it

($\Rightarrow$): A better idea–"dovetailing":

For i = 0, 1, 2, 3, ... :

At stage i, run the recognizer for i steps on each of the first i strings in Σ^*, output any that are accepted.

Encoding things



$\langle G \rangle =$

`(1,2,3,4)((1,2),(2,3),(3,1),(1,4))`

Σ = ?

CFG G = (V, Σ, R, S) ; <G> = ((S,A,B,...),(a,b,...), (S→aA, S→b, A→cAb, ...),S)

or <G> = ((A_0, A_1, ...),(a_0, a_1, ...), ($A_0 \to a_0 A_1$, $A_0 \to a_1$, $A_1 \to a_2 A_1 a_1$, ...), A_0)

DFA D = (Q, Σ, δ, q_0, F); <D> = (...)

TM M = (Q, Σ, Γ, δ, q_0, q_a, q_r); <M> = (...)

...

Decidability

Recall: L *decidable* means there is a TM recognizing L *that always halts*.

Example:

"The acceptance problems for DFAs"

A_{DFA} = { <D,w> | D is a DFA & w $\in$ L(D) }

Some Decidable Languages

The following are decidable:

A_{DFA} = { <D,w> | D is a DFA & w $\in$ L(D) }

pf: simulate D on w

A_{NFA} = { <N,w> | N is an NFA & w $\in$ L(N) }

pf: convert N to a DFA, then use previous as a subroutine

A_{REX} = { <R,w> | R is a regular expr & w $\in$ L(R)}

pf: convert R to an NFA, then use previous as a subroutine

$EMPTY_{DFA}$ = {<D> | D is a DFA and L(D) = ∅ }

pf: is there no path from start state to any final state?

EQ_{DFA} = { <A,B> | A & B are DFAs s.t. L(A)= L(B) }

pf: equal iff L(A)⊕L(B) = ∅, and $x \oplus y = (x \cap y^c) \cup (x^c \cap y)$, and regular sets are closed under ∪, ∩, complement

A_{CFG} = { <G,w> | ... }

pf: see book

$EMPTY_{CFG}$ = { <G> | ... }

pf: see book

EQ_{CFG} = { <A,B> | A & B are CFGs s.t. L(A) = L(B) }

This is *NOT* decidable