CSE 431 Spring 2012
Assignment #3

Due: Friday, April 20, 2012

Reading assignment: Read Chapter 5 of Sipser's text. We will cover section 5.3 before we cover computation histories in section 5.1.

Problems:

1. Show that the set of complex numbers,

   $QUADRATIC\text{-}ROOT = \{x \in \mathbb{C} \mid$ there are integers $a \neq 0$, b, and c such that $ax^2+bx+c = 0\}$

   is countable.

2. Suppose that $A \subseteq \{\langle M \rangle \mid M$ is a decider TM$\}$ and that A is Turing-recognizable. (That is, A only contains descriptions of TMs that are deciders but it might not accept all such descriptions.)
   Prove that there is a decidable language D such that $L(M) \neq D$ for any M with $\langle M \rangle \in A$. (Intuitively, this means that one couldn't come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for A.)

3. Let $T = \{\langle M \rangle \mid M$ is a TM that accepts w^R whenever it accepts $w\}$. Show that T is undecidable.

4. Sipser's text (either edition), Problem 5.13.

5. Sipser's text (1st edition problem 5.10; 2nd edition problem 5.24.

6. (Bonus) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the *busy beaver function* $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of k, consider all k-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.