CSE 431 – Theory of Computation. Spring, 2014. Instructor: James R. Lee

ASSIGNMENT 0. Due Thursday, April 10th, in class (or via email to cse431-staff@cs)

1. Prove that if A and B are countable, then so is $A \cup B$.

2. Suppose that A is countable and $f : A \to B$ is any function. Prove that $f(A)$ is countable or finite, where

$$f(A) = \{y \in B : f(x) = y \text{ for some } x \in A\}$$

3. Prove that if A is countable and $f : A \to B$ is surjective, then B is countable or finite.

4. You may recall that if Σ is a set, then Σ^* denotes all the finite-length strings over Σ. For instance, if $\Sigma = \{a, b, c\}$ then $\Sigma^* = \{\lambda, a, b, c, aa, ab, ac, ba, bb, bc, aaa, aab, aac, aba, abb, \dots\}$. Here, λ denotes the empty string.

   (i) Prove that if Σ is finite then Σ^* is countable.
   (ii) Prove that if Σ is countable then Σ^* is countable.

5. Suppose that Σ is a finite alphabet (with at least two symbols). Consider the set of all **infinite strings** over Σ. These are strings $s_1 s_2 s_3 \dots \dots$ such that each symbols $s_i \in \Sigma$. Prove that the set of all such infinite strings is uncountable.

OPTIONAL PROBLEM (You may do this problem for extra credit, OR you can do it instead of the first five problems!)

A simple question: Does there exist a way to draw an uncountable number of 'B's in the plane such that none of them intersect? (Here you may assume that the line segment and the two "bumps" are simply one-dimensional curves. The copies can be rotated or scaled.) If so, give a recipe for drawing them! If not, provide a proof that no such drawing exists.