CSE 431 – Theory of Computation. Spring, 2014. Instructor: James R. Lee

ASSIGNMENT 7. Due Tuesday, June 3rd, in class (or via email to cse431-staff@cs before class starts)

There are three problems + one extra credit.

1. The Japanese game Gomoku is played on a 19-by-19 board. There are two players: One has black stones and the other has white stones. The players alternately place stones until the board is full or someone wins. A player wins when they get five of their stones in a row, either horizontally, vertically, or diagonally. Consider the game **Asym-Gomoku** which is played on an $n \times n$ board according to the same rules.

   Define the language
   $GM = \{ \langle P \rangle : P$ is a position in Asym-Gomoku where White has a winning strategy $\}$

   A position is simply a state of the game board, together with which player moves next. A winning strategy means that there is a way for White to force a win.

   Prove that $GM \in \mathbf{PSPACE}$.

2. Recall that EXPTIME $= \bigcup_k \mathrm{TIME}\left(2^{n^k}\right)$. Let NEXPTIME = $\bigcup_k \mathrm{NTIME}\left(2^{n^k}\right)$ be the class of languages decidable on a non-deterministic TM with exponential time. Your goal in this problem is to show that if EXPTIME ≠ NEXPTIME, then P ≠ NP.

   To accomplish this, it will help to use the function

   $$\mathrm{pad} : \Sigma^* \times \mathbb{N} \to \Sigma^* \#^*$$

   defined by $\mathrm{pad}(s, l) = s\#^j$ where $j = \max(0, l - \mathrm{length}(s))$. In other words, the function pad adds enough # characters to the end of s so that it has length exactly l (and it just returns s if $\mathrm{length}(s) > l$).

   For a language A and a function $f : \mathbb{N} \to \mathbb{N}$, we define a new language
   $$\mathrm{pad}(A, f(n)) = \left\{\mathrm{pad}\left(s, f\left(\mathrm{length}(s)\right)\right) : s \in A\right\}$$

   a) Prove that if $A \in TIME(n^{10})$, then $\mathrm{pad}(A, n^2) \in TIME(n^5)$.

   b) Prove that if EXPTIME ≠ NEXPTIME, then P ≠ NP.

3. Let A be the language of properly nested parentheses. For example, (()) and (()(()))() are in A, but)(is not. Show that $A \in L$.

   (Note that L is the class of languages that can be decided in $O(\log n)$ space. We will see the definition on Tuesday, 5/27.)

OPTIONAL PROBLEM (You may do this problem for extra credit, OR you can do it instead of the first three problems!)

Consider the language

$$\text{ACYCLIC} = \{\langle G \rangle : G \text{ is an undirected graph with no cycles } \}$$

(Note that G could be disconnected.) Prove that $\text{ACYCLIC} \in L$.