CSE 431 Spring 2015
Assignment #1

Due: Friday, April 9, 2015

Reading assignment: Read Chapter 3 of Sipser's text.

Problems:

1. A **Turing machine with stay put instead of left** is similar to an ordinary Turing machine, but the transition function has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$. At each point, the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is *not* equivalent to the usual version. What class of languages do these machines recognize? (Note that you can use without proof properties of languages proved in CSE 311.)

2. Explain why the following is not a description of a legitimate Turing machine.

   M_{bad}="On input $\langle p \rangle$, a polynomial over variables $x_1, \ldots, x_k$:

   (a) Try all possible settings of $x_1, \ldots, x_n$ to integer values.

   (b) Evaluate p on all of these settings.

   (c) If any of these settings evaluates to 0, *accept*; otherwise, *reject*."

3. Give a Turing machine diagram for a Turing machine that on input a string $x \in \{0, 1\}^*$ halts (accepts) with its head on the left end of the tape containing the string $x' \in \{0, 1\}^*$ at the left end (and blank otherwise) where x' is the successor string of x in lexicographic order; i.e. the next string in the sequence $\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots$ in which the strings are listed in order of increasing length with ties broken by their corresponding integer value. (Briefly document your TM.)

4. Turing in his paper said that the 2-dimensional nature of the paper is not essential. In this question you will show why that is the case:
   A Turing machine with a 2-dimensional tape is like a 1-tape TM except that it marked with an infinite 2-dimensional grid of cells that are all blank, except for the input which is given in the cells starting with the cell under the read/write head and continuing with the sequence of cells immediately to the right. Additional changes are that

   - the transition function δ, is $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, U, D\}$ where U and D indicate moves *up* and *down* one cell.
   - there is no end of the tape.

   Give an implementation level description of how an ordinary 1-dimensional Turing machine can simulate a 2-dimensional one; that is, the 1-dimensional TM should accept, reject, or run forever on exactly the same set of inputs as the 2-dimensional one does.

5. (Extra credit) A 2-PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a machine that is like a finite state machine with **two stacks** which it can access at the same time. The *input* alphabet $\Sigma \subset \Gamma$, which is the *stack* alphabet. F is a set of final states. The transition function

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\})^2 \to Q \times (\Gamma \cup \{\varepsilon\})^2.$$

The interpretation is that on reading an input symbol (or nothing) and reading and popping (or ignoring) the top symbol on each stack, it moves to a new state and changes the top of the two stacks (either by pushing on a new symbol or nothing onto each stack). Show that for any Turing machine there is a 2-PDA that accepts precisely the same set of inputs.

6. (Extra credit) Do the same for a machine like the above that has one queue instead of two stacks.