## CSE 431 Spring 2017 Assignment #2

Due: Friday, April 14, 2017

**Reading assignment:** Read Chapter 4 of Sipser's text.

## **Problems:**

- 1. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)
- 2. Use the result of question 1 to show that any infinite Turing-recognizable language contains an infinite decidable subset.
- 3. Let  $ALL_{DFA} = \{ \langle M \rangle \mid M \text{ is a DFA with alphabet } \Sigma \text{ and } L(M) = \Sigma^* \}$ . Prove that  $ALL_{DFA}$  is decidable.
- 4. Show that the decidable languages are closed under the concatenation, intersection, and star operations.
- 5. Show that the Turing-recognizable languages are closed under the concatenation, intersection, and star operations.
- 6. (Bonus) Let C be a language. Prove that C is Turing-recognizable iff there is a decidable language D such that  $C = \{x \mid \exists y \text{ such that } \langle x, y \rangle \in D\}$ .