CSE 431 Spring 2017
Assignment #6

Due: Friday, May 19, 2017

Reading assignment: Read section 7.5 and sections 8.1-8.2.

Problems:

1. Show that if $P = NP$ then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.

2. Let $U = \{\langle M, x, 1^t \rangle \mid M \text{ is an NTM that accepts input } x \text{ within } t \text{ steps}\}$. Show that U is NP-complete.

3. Let ϕ be a 3CNF-formula. An NOTALLEQUAL assignment to the variables of ϕ is one where each clause contains two literals with different truth values. In other words a NOTALLEQUAL assignment satisfies ϕ but does not set all three literals to true in any clause.

   (a) Show that the negation of a NOTALLEQUAL assignment for ϕ is also a NOTALLEQUAL assignment for ϕ.

   (b) Let *NAESAT* be the set of all 3CNF formulas ϕ that have a NOTALLEQUAL assignment. Prove that *NAESAT* is NP-complete. For the hardness part use a reduction from 3SAT.
   (Hint: Show that the function that replaces each clause C_i of ϕ of the form $(y_1 \vee y_2 \vee y_3)$ where y_1, y_2, y_3 are literals by the two clauses $(y_1 \vee y_2 \vee z_i)$ and $(\overline{z_i} \vee y_3 \vee w)$ where w is a single new variable for all clauses and there is one z_i variable per original clause.)

4. A *cut* in an undirected graph G is a partition of the vertices V of G into two disjoint parts S and T with $V = S \cup T$. The *size* of the cut (S, T) is the number of edges that cross between S and T. Define

$$MAXCUT = \{\langle G, k \rangle \mid G \text{ has a cut of size } \geq k\}.$$

   Show that $MAXCUT$ is NP-complete. For the hardness part use the fact that $NAESAT$ is NP-complete.
   (Hint: For each variable x in an m-clause 3CNF formula, have $3m$ vertices for x and $3m$ vertices for $\overline{x}$ for a total of $6m$ vertices in the graph. Join each pair of nodes with the same variable label, but different signs by an edge. For each clause, consider a separate vertex of every possible literal label to be dedicated to that clause. Among those literals, add edges that join the 3 literals that actually appear in the clause to form a triangle. What value of k should you use? Prove that this works.)

5. Let $01ROOT = \{\langle p \rangle \mid p$ is a polynomial in n variables with integer coefficients such that $p(x_1, \ldots, x_n) = 0$ for some assignment $(x_1, \ldots, x_n) \in \{0, 1\}^n\}$.

   (a) Show that $01ROOT \in NP$.

   (b) Show that $3SAT \leq_P 01ROOT$. (HINT: First figure out how to convert each clause into a polynomial that evaluates to 0 iff the clause is satisfied. Then create a polynomial q that evaluates to 0 if and only if all of its inputs are 0. Finally, figure out how to combine the individual polynomials for the clauses using the polynomial q.

6. (Extra credit) Recall that a 2-CNF formula is a CNF formula in which each clause has 2 literals and that 2-$SAT = \{\langle \phi \rangle \mid \phi$ is a satisfiable 2-CNF formula$\}$. Show that $2SAT \in P$.