CSE 431 Fall 2019
Assignment #2

Due: Thursday, October 10, 2019, 11:59 PM

Reading assignment: Read Chapter 4 of Sipser's text.

Problems:

1. (20 points) Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case where it is infinite.)

2. (10 points) Use the result of question 1 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

3. (30 points) Suppose that A and B are decidable languages. Prove that the following languages are also decidable. (The definitions here are from Chapter 1 and are included for convenience.)

   (a) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

   (b) $AB = \{x \mid \exists y \in A \text{ and } z \in B \text{ such that } x = yz\}$.

   (c) $A^* = \{x \mid \exists k \geq 0 \text{ and } y_1, \ldots, y_k \in A \text{ such that } x = y_1 \cdots y_k\}$.

4. (30 points) Suppose that A and B are Turing-recognizable languages. Prove that (a) $A \cap B$, (b) AB, and (c) A^* are all also Turing-recognizable.

5. (Extra credit) Let C be a language. Prove that C is Turing-recognizable if and only if there is a decidable language D such that $C = \{x \mid \exists y \text{ such that } \langle x, y \rangle \in D\}$.