# CSE 431 Fall 2019
# Assignment #3

Due: Thursday, October 17, 2019 11:59 PM

**Reading assignment:** Read Chapter 5 of Sipser's text. We will cover section 5.3 before we cover computation histories in section 5.1.

**Problems:**

1. (20 points) Define $ALL_{DFA} = \{\langle M\rangle \mid M \text{ is a DFA with alphabet } \Sigma \text{ and } L(M) = \Sigma^*\}$. Prove that $ALL_{DFA}$ is decidable.

2. (20 points) Define

   $$INFINITE_{CFG} = \{\langle G\rangle \mid \text{ the language that context-free grammar } G \text{ generates is infinite}\}.$$

   Prove that $INFINITE_{CFG}$ is decidable.

3. (20 points) A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

4. (20 points) Define $SUBSET_{TM} = \{\langle M_1, M_2\rangle \mid M_1, M_2 \text{ are TMs and } L(M_1) \subseteq L(M_2)\}$. Prove that $SUBSET_{TM}$ is undecidable.

5. (20 points) Suppose that $A \subseteq \{\langle M\rangle \mid M \text{ is a decider TM}\}$ and that $A$ is Turing-recognizable. (That is, $A$ only contains descriptions of TMs that are deciders but it might not contain all such descriptions.)
   Prove that there is a decidable language $D$ such that $L(M) \neq D$ for every $M$ with $\langle M\rangle \in A$. (Intuitively, this means that one couldn't come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for $A$.)

6. (Extra Credit) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of $k$, consider all $k$-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that $BB$ is not a computable function.