# CSE 431 Fall 2019
# Assignment #7

Due: Monday, Nov 25, 2019 8:59 PM

**Reading assignment:** Read Sections 8.1-8.5.

**Problems:**

1. Define

$$THIRD\text{-}CLIQUE = \{\langle G\rangle \mid G \text{ is an undirected graph with } n = 3\ell \text{ nodes for some integer } \ell \text{ and } G \text{ has a clique on } \ell \text{ nodes}\}.$$

   Prove that $THIRD\text{-}CLIQUE$ is $NP$-complete.
   (Hint: Use the fact the $CLIQUE$ is $NP$-complete.)

2. This problem is inspired by the single-player game *Minesweeper*, generalized to an arbitrary graph. Minesweeper begins with an undirected graph $G$ in which each node either contains a single, hidden mine or is empty. The player chooses nodes, one by one. If the player chooses a node containing a mine, the player loses. If the player chooses an empty node, the player learns the number of neighboring nodes containing mines. The player wins if and when the player has chosen all the empty nodes. (In the actual game it suffices for the player to learn the number of neighboring nodes associated with each empty node.)

   We are interested in the related problem of *Mine-Consistency* in which the input is an undirected graph $G$ together with numbers for some of $G$'s nodes. The goal is to determine whether there is a placement of mines on the remaining nodes so that any node $u$ numbered $k$ has exactly $k$ neighboring nodes containing mines. Formulate *Mine-Consistency* as a language, and prove that it is $NP$-complete.

   Hint: One possibility is a reduction from 3SAT. The reduction from 3SAT to SUBSET-SUM in the text might inspire you in the right direction.

3. Let $EQ_{REX} = \{\langle R, S\rangle \mid R \text{ and } S \text{ are equivalent regular expressions}\}$. Show that $EQ_{REX} \in PSPACE$.

4. Prove that if every $NP$-hard language is $PSPACE$-hard then $NP = PSPACE$.

5. Let $A_{LBA} = \{\langle M, w\rangle \mid M \text{ is an LBA that accepts input } w\}$. Show that $A_{LBA}$ is PSPACE-complete.

6. (Extra Credit) Define the function $\textsc{Majority}$:$\{0,1\}^* \to \{0,1\}$ by $\textsc{Majority}(x) = 1$ iff $\geq 1/2$ the bits in $x$ are 1. Let $C_{\textsc{Majority}} : \mathbb{N} \to \mathbb{N}$ be the smallest function such that for every $n \in \mathbb{N}$, there is a circuit $C_n$ of size at most $C_{\textsc{Majority}}(n)$ that computes $\textsc{Majority}$ on all strings $x \in \{0,1\}^n$.

   (a) Show that $C_{\textsc{Majority}}(n)$ is $O(n^2)$.

   (b) Show how to compute the sum of the bits of $x$ using divide and conquer and use this to show that $C_{\textsc{Majority}}(n)$ is $O(n \log n)$.

   (c) Find another clever idea to show that $C_{\textsc{Majority}}(n)$ is $O(n)$.