CSE 431 Winter 2022
Assignment #1

Due: Thursday January 13, 2022, 11:59 PM

Reading assignment: Read Chapter 3 of Sipser's text.

Problems:

0. (5 points) Confirm that you have read the course homework policy on the course webpage .

1. (25 points) Give an implementation-level description of a Turing machine M that decides the language $\{x\#y\#z \mid x, y, z \in \{0,1\}^*,\ |x| = |y| = |z|,\ z = x \oplus y\}$. (Recall that $x \oplus y$ is the string whose bits are the XOR of the corresponding bits in x and y.)

2. (35 points) Give a Turing machine diagram for a Turing machine that on input a string $x \in \{0,1\}^*$ halts (accepts) with its head on the left end of the tape containing the string $x' \in \{0,1\}^*$ at the left end (and blank otherwise) where x' is the successor string of x in lexicographic order; i.e. the next string in the sequence $\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots$ in which the strings are listed in order of increasing length with ties broken by their corresponding integer value. (Briefly document your TM by giving an implementation level description of your algorithm with the names of the states where each part of the implementation is done indicated in parentheses.)

3. (35 points) Turing in his paper said that the 2-dimensional nature of the paper used by a computer is not essential. In this question you will show why it doesn't matter:
A Turing machine with a 2-dimensional tape is like a 1-tape TM except that it marked with an infinite 2-dimensional grid of cells that are all blank, except for the input. Additional changes are that

   - the input is given in a finite sequence of cells starting with the cell under the read/write head and moving to the right, ending just before the first blank cell encountered. (This is just the way it would be if the cells under and to the right of the read/write head were viewed as the 1-way infinite tape of an ordinary Turing machine.)
   - the transition function δ, is $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, U, D\}$ where U and D indicate moves *up* and *down* one cell.
   - the tape is infinite in all 4 directions.

   Give an implementation-level description of how an ordinary 1-dimensional Turing machine can simulate a 2-dimensional one; that is, the 1-dimensional TM should accept, reject, or run forever on exactly the same set of inputs as the 2-dimensional one does.

4. (Extra credit) A 2-PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is a machine that is like a finite state machine with **two stacks** which it can access at the same time. The *input* alphabet $\Sigma \subset \Gamma$, which is the *stack* alphabet. F is a set of final states. The transition function

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\})^2 \to Q \times (\Gamma \cup \{\varepsilon\})^2.$$

The interpretation is that on reading an input symbol (or nothing) and reading and popping (or ignoring) the top symbol on each stack, it moves to a new state and changes the top of the two stacks (either by pushing on a new symbol or nothing onto each stack). The $\Sigma \cup \{\varepsilon\}$ portion refers to the first symbol of the input that is being read (or ignored in the case of ε). Γ is the alphabet of stack symbols. $\Gamma \cup \{\varepsilon\}$ refers to what happens to the top of each stack - on the left side it is ε if nothing is looked at and otherwise it is the value of the top symbol that has to be read (and will be popped) when the transition takes place; on the right side it refers to what will be pushed on each stack.

A 2-PDA M accepts input x iff when M is started in state q_0 with both of its stacks empty, there is a series of applications of δ that lets M reach some state in F. Show that for any Turing machine there is a 2-PDA that accepts precisely the same set of inputs.

5. (Extra credit) Do the same for a machine like the above that has one queue instead of two stacks.