CSE 431 Winter 2022
Assignment #2

Due: Thursday January 20, 2022, 11:59 PM

Reading assignment: Read Chapter 4 of Sipser's textbook.

Problems:

1. (20 points) Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order.
   (Hint: Handle the case where the language is finite separately from the case where it is infinite.)

2. (10 points) Use the result of question 1 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

3. (10 points) Prove that the language
   $ALL_{DFA} = \{\langle M \rangle \mid M$ is a DFA with alphabet Σ and $L(M) = \Sigma^*\}$ is decidable.

4. (30 points) Suppose that A and B are decidable languages. Prove that the following languages are also decidable. (The definitions of the latter two are from Chapter 1 and all are included for convenience.)

   (a) $A \cap B = \{x \mid x \in A$ and $x \in B\}$.

   (b) $AB = \{x \mid \exists y \in A$ and $z \in B$ such that $x = yz\}$.

   (c) $A^* = \{x \mid \exists k \geq 0$ and $y_1, \ldots, y_k \in A$ such that $x = y_1 \cdots y_k\}$.

5. (30 points) Suppose that A and B are Turing-recognizable languages. Prove that the following languages are also Turing-recognizable:

   (a) AB.

   (b) $Pref(A) = \{x \mid \exists y$ with $xy \in A\}$, the set of all prefixes of strings in A.

6. (Extra credit) Let C be a language. Prove that C is Turing-recognizable if and only if there is a decidable language D such that $C = \{x \mid \exists y$ such that $\langle x, y \rangle \in D\}$.