Introduction to Database Systems
CSE 444

Lecture 18: Relational Algebra

Outline

- Motivation and sets v.s. bags
- Relational Algebra
- Translation from SQL to the Relational Algebra

- Read Sections 2.4, 5.1, and 5.2
  - [Old edition: 5.1 through 5.4]
  - These book sections go over relational operators

The WHAT and the HOW

- In SQL, we write WHAT we want to get from the data
- The database system needs to figure out HOW to get the data we want
- The passage from WHAT to HOW goes through the **Relational Algebra**

SQL = WHAT

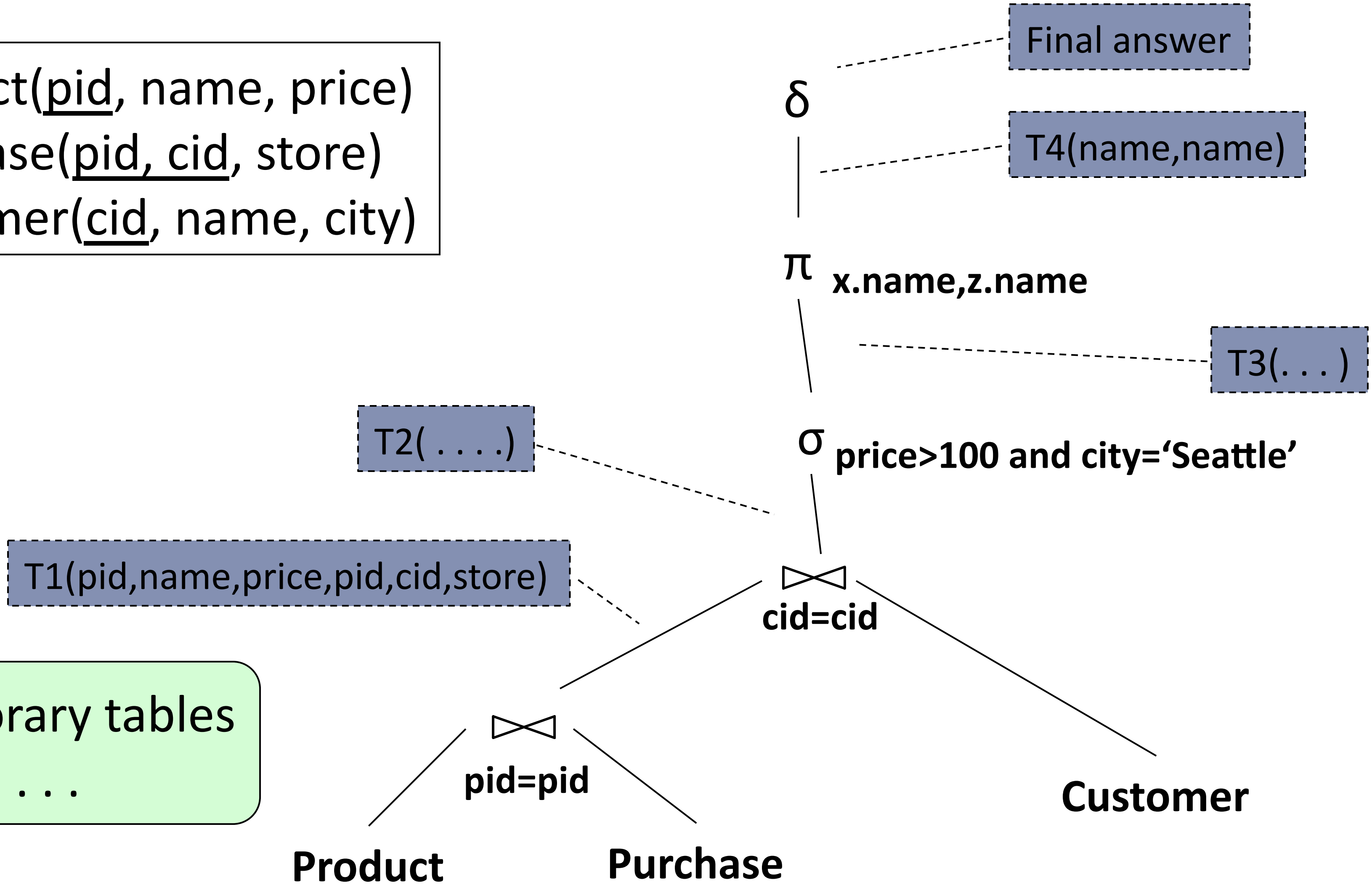
Product(pid, name, price)
Purchase(pid, cid, store)
Customer(cid, name, city)

```
SELECT DISTINCT x.name, z.name
FROM Product x, Purchase y, Customer z
WHERE x.pid = y.pid and y.cid = z.cid and
          x.price > 100 and z.city = 'Seattle'
```

It's clear WHAT we want, unclear HOW to get it

Relational Algebra = HOW

Product(<u>pid</u>, name, price)
Purchase(<u>pid, cid</u>, store)
Customer(<u>cid</u>, name, city)



Temporary tables
T1, T2, . . .

Relational Algebra = HOW

The order is now clearly specified:

- Iterate over PRODUCT…
- …join with PURCHASE…
- …join with CUSTOMER…
- …select tuples with Price>100 and City='Seattle'…
- …eliminate duplicates…
- …and that's the final answer !

Relations

- A relation is a set of tuples
  - Sets: {a,b,c}, {a,d,e,f}, { }, . . .
- But, commercial DBMS's implement relations that are bags rather than sets
  - Bags: {a, a, b, c}, {b, b, b, b, b}, . . .

Sets v.s. Bags

Relational Algebra has two flavors:

- Over sets: theoretically elegant but limited
- Over bags: needed for SQL queries + more efficient
  - Example: Compute average price of all products

We discuss set semantics

- We mention bag semantics only where needed

Outline

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Relational Algebra

- **Query language** associated with relational model
- Queries specified in an operational manner
  - A query gives a step-by-step procedure
- Relational operators
  - Take one or two relation instances as argument
  - Return one relation instance as result
  - Easy to compose into relational algebra expressions

Relational Algebra (1/3)

Five basic operators:

- Union (∪) and Set difference (–)
- Selection: : $\sigma_{\text{condition}}(S)$
  - Condition is Boolean combination (∧,∨) of terms
  - Term is: attribute op constant, attr. op attr.
  - Op is: <, <=, =, ≠, >=, or >
- Projection: $\pi_{\text{list-of-attributes}}(S)$
- Cross-product or cartesian product (×)

Relational Algebra (2/3)

Derived or auxiliary operators:

- Intersection ($\cap$), Division (R/S)
- Join: $R \bowtie_\theta S = \sigma_\theta(R \times S)$
- Variations of joins
  - Natural, equijoin, theta-join
  - Outer join and semi-join
- Rename $\rho_{B1,\ldots,Bn}(S)$

Relational Algebra (3/3)

Extensions for bags

- Duplicate elimination: δ
- Group by: γ [Same symbol as aggregation]
  - Partitions tuples of a relation into "groups"
- Sorting: τ

Other extensions

- Aggregation: γ (min, max, sum, average, count)

Union and Difference

- R1 ∪ R2
- Example:
  - ActiveEmployees ∪ RetiredEmployees

- R1 – R2
- Example:
  - AllEmployees – RetiredEmployees

Be careful when applying to bags!

What about Intersection?

- It is a derived operator
- $R1 \cap R2 = R1 - (R1 - R2)$
- Also expressed as a join (will see later)
- Example
  - UnionizedEmployees ∩ RetiredEmployees

Relational Algebra (1/3)

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Selection

- Returns all tuples that satisfy a condition
- Notation: $\sigma_c(R)$
- Examples
  - $\sigma_{Salary > 40000}$ (Employee)
  - $\sigma_{name = \text{“Smith”}}$ (Employee)
- The condition c can be
  - Boolean combination ($\wedge$, $\vee$) of terms
  - Term is: attribute op constant, attr. op attr.
  - Op is: <, <=, =, ≠, >=, or >

Maps to the WHERE clause in SQL!

Selection example

SSN	Name	Salary
1234545	John	20000
5423341	Smith	60000
4352342	Fred	50000

$\sigma_{\text{Salary} > 40000}$ (Employee)

SSN	Name	Salary
5423341	Smith	60000
4352342	Fred	50000

x

Projection

- Eliminates columns
- Notation: $\pi_{A1,\ldots,An}(R)$
- Example: project social-security number and names:
  - $\pi_{SSN,\ Name}$ (Employee)
  - Output schema: Answer(SSN, Name)

Semantics differs over set or over bags

Projection example

Set semantics: duplicate elimination automatic

SSN	Name	Salary
1234545	John	20000
5423341	John	60000
4352342	John	20000

$\pi_{\text{Name,Salary}}$ (Employee)

Name	Salary
John	20000
John	60000

x

Example

Bag semantics: no duplicate elimination; need explicit δ

SSN	Name	Salary
1234545	John	20000
5423341	John	60000
4352342	John	20000

$\pi_{\text{Name,Salary}}$ (Employee)

Name	Salary
John	20000
John	60000
John	20000

x

Selection & Projection Examples

Patient

no	name	zip	disease
1	p1	98125	flu
2	p2	98125	heart
3	p3	98120	lung
4	p4	98120	heart

$\pi_{zip,disease}$(Patient)

zip	disease
98125	flu
98125	heart
98120	lung
98120	heart

$\sigma_{disease='heart'}$(Patient)

no	name	zip	disease
2	p2	98125	heart
4	p4	98120	heart

π_{zip} ($\sigma_{disease='heart'}$(Patient))

zip
98120
98125

Relational Algebra (1/3)

Five basic operators:

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  - Term is: attribute op constant, attr. op attr.
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- Cross-product or cartesian product (×)

Cartesian Product

- Each tuple in R1 with each tuple in R2
- Notation: R1 × R2
- Example:
  - Employee × Dependents
- Rare in practice; mainly used to express joins

Cartesian Product Example

Employee

Name	SSN
John	999999999
Tony	777777777

Dependents

EmployeeSSN	Dname
999999999	Emily
777777777	Joe

Employee x Dependents

Name	SSN	EmployeeSSN	Dname
John	999999999	999999999	Emily
John	999999999	777777777	Joe
Tony	777777777	999999999	Emily
Tony	777777777	777777777	Joe

Relational Algebra (2/3)

Derived or auxiliary operators:

- Intersection ($\cap$), Division (R/S)
- Join: $R \bowtie_\theta S = \sigma_\theta(R \times S)$
- Variations of joins
  - Natural, equijoin, theta-join
  - Outer join and semi-join
- Rename $\rho_{B1,\ldots,Bn}(S)$

Renaming

- Changes the schema, not the instance
- Notation: $\rho_{B1,\ldots,Bn}(R)$
- Example:
  - $\rho_{LastName,\ SocSecNo}$ (Employee)
  - Output schema: Answer(LastName, SocSecNo)

Renaming Example

Employee

Name	SSN
John	999999999
Tony	777777777

$$\rho_{LastName,\ SocSocNo}\ (\text{Employee})$$

Employee

LastName	SocSecNo
John	999999999
Tony	777777777

Relational Algebra (2/3)

Derived or auxiliary operators:

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Different Types of Join

- **Theta-join**: $R \bowtie_\theta S = \sigma_\theta(R \times S)$
  - Join of R and S with a join condition θ
  - Cross-product followed by selection θ

- **Equijoin**: $R \bowtie_\theta S = \pi_A (\sigma_\theta(R \times S))$
  - Join condition θ consists only of equalities
  - Projection π_A drops all redundant attributes

- **Natural join**: $R \bowtie S = \pi_A (\sigma_\theta(R \times S))$
  - Equijoin
  - Equality on all fields with same name in R and in S

Theta-Join Example

AnonPatient P

age	zip	disease
54	98125	heart
20	98120	flu

AnonJob J

job	age	zip
lawyer	54	98125
cashier	20	98120

$$P \bowtie_{P.age=J.age \,\wedge\, P.zip=J.zip \,\wedge\, P.age < 50} J$$

P.age	P.zip	disease	job	J.age	J.zip
20	98120	flu	cashier	20	98120

Equijoin Example

AnonPatient P

age	zip	disease
54	98125	heart
20	98120	flu

AnonJob J

job	age	zip
lawyer	54	98125
cashier	20	98120

$P \bowtie_{P.age=J.age} J$

age	P.zip	disease	job	J.zip
54	98125	heart	lawyer	98125
20	98120	flu	cashier	98120

Natural Join Example

AnonPatient P

age	zip	disease
54	98125	heart
20	98120	flu

AnonJob J

job	age	zip
lawyer	54	98125
cashier	20	98120

$P \bowtie J$

age	P.zip	disease	job
54	98125	heart	lawyer
20	98120	flu	cashier

So which join is it ?

- When we write R ⋈ S we usually mean an equijoin, but we often omit the equality predicate when it is clear from the context

More Joins

- **Outer join**
  - Include tuples with no matches in the output
  - Use NULL values for missing attributes

- Variants
  - Left outer join
  - Right outer join
  - Full outer join

Outer Join Example

AnonPatient P

age	zip	disease
54	98125	heart
20	98120	flu
33	98120	lung

AnonJob J

job	age	zip
lawyer	54	98125
cashier	20	98120

P ⟕ V

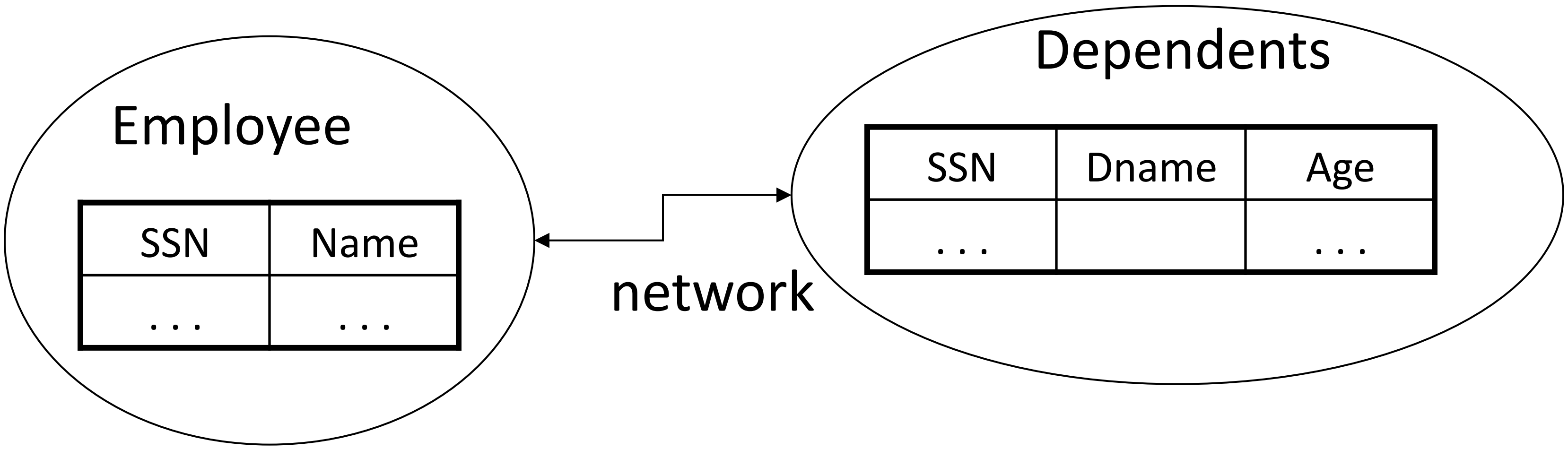
age	zip	disease	job
54	98125	heart	lawyer
20	98120	flu	cashier
33	98120	lung	null

Semijoin

- $R \ltimes S = \Pi_{A1,\ldots,An} (R \bowtie S)$
- Where $A_1, \ldots, A_n$ are the attributes in R
- Example:
  - Employee $\ltimes$ Dependents

Semijoins in Distributed Databases

- Semijoins are used in distributed databases



$$\text{Employee} \bowtie_{ssn=ssn} (\sigma_{age>71}(\text{Dependents}))$$

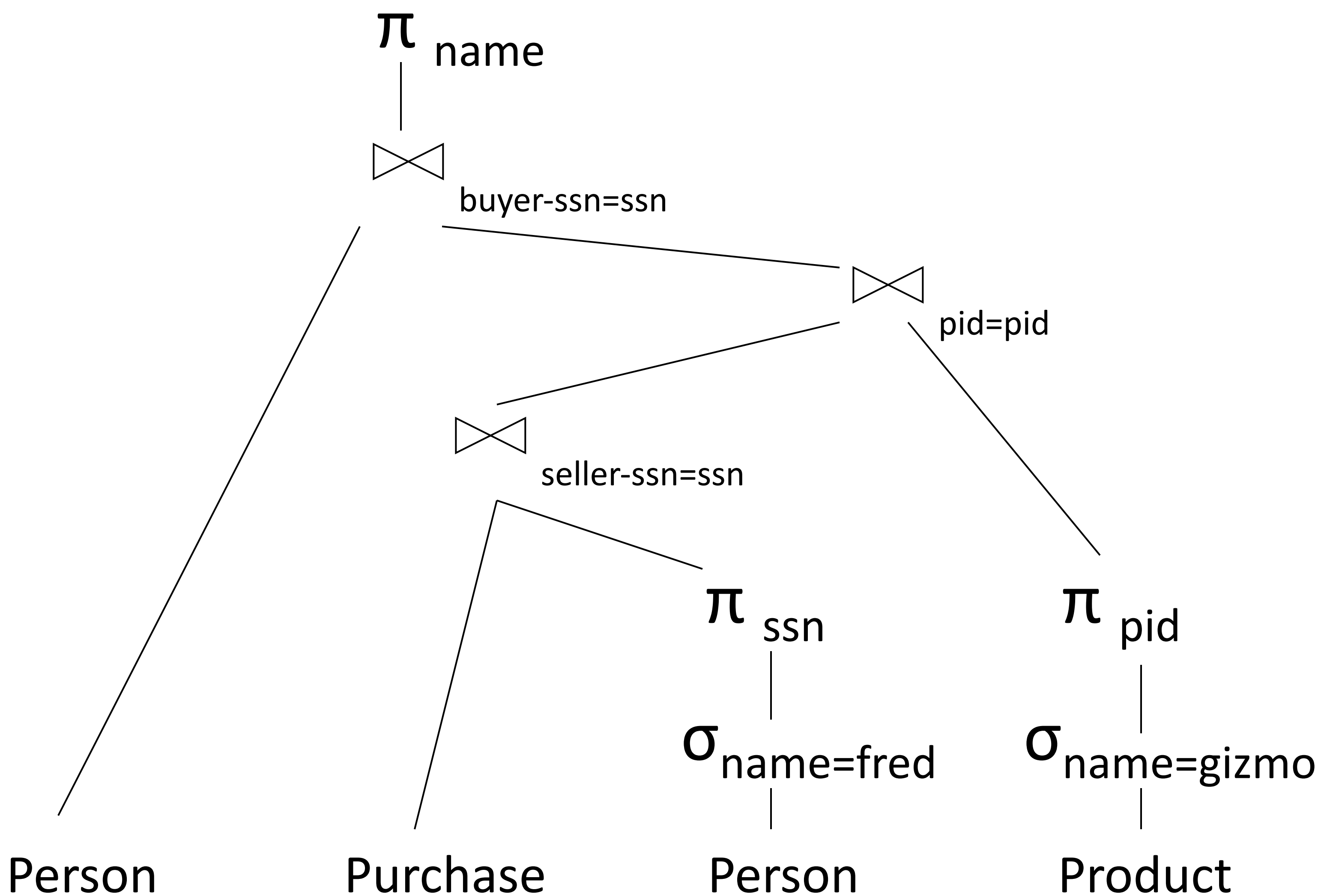
$$T = \pi_{SSN}(\sigma_{age>71}(\text{Dependents}))$$

$$R = \text{Employee} \ltimes T$$

$$\text{Answer} = R \bowtie \text{Dependents}$$

x

Complex RA Expressions



Example of Algebra Queries

Q1: Jobs of patients who have heart disease

$\pi_{job}(\text{AnonJob} \bowtie (\sigma_{disease=\text{'heart'}}\ (\text{AnonPatient}))$

More Examples

Supplier(sno,sname,scity,sstate)

Part(pno,pname,psize,pcolor)

Supply(sno,pno,qty,price)

Q2: Name of supplier of parts with size greater than 10

$\pi_{sname}(\text{Supplier} \bowtie \text{Supply} \bowtie (\sigma_{psize>10}\ (\text{Part}))$

Q3: Name of supplier of red parts or parts with size greater than 10

$\pi_{sname}(\text{Supplier} \bowtie \text{Supply} \bowtie (\sigma_{psize>10}\ (\text{Part}) \cup \sigma_{pcolor=\text{'red'}}\ (\text{Part})\ )\ )$

RA Expressions v.s. Programs

- An Algebra Expression is like a program
  - Several operations
  - Strictly specified order

- But Algebra expressions have limitations

RA and Transitive Closure

- Cannot compute "transitive closure"

Name1	Name2	Relationship
Fred	Mary	Father
Mary	Joe	Cousin
Mary	Bill	Spouse
Nancy	Lou	Sister

- Find all direct and indirect relatives of Fred
- Cannot express in RA!!! Need to write Java program

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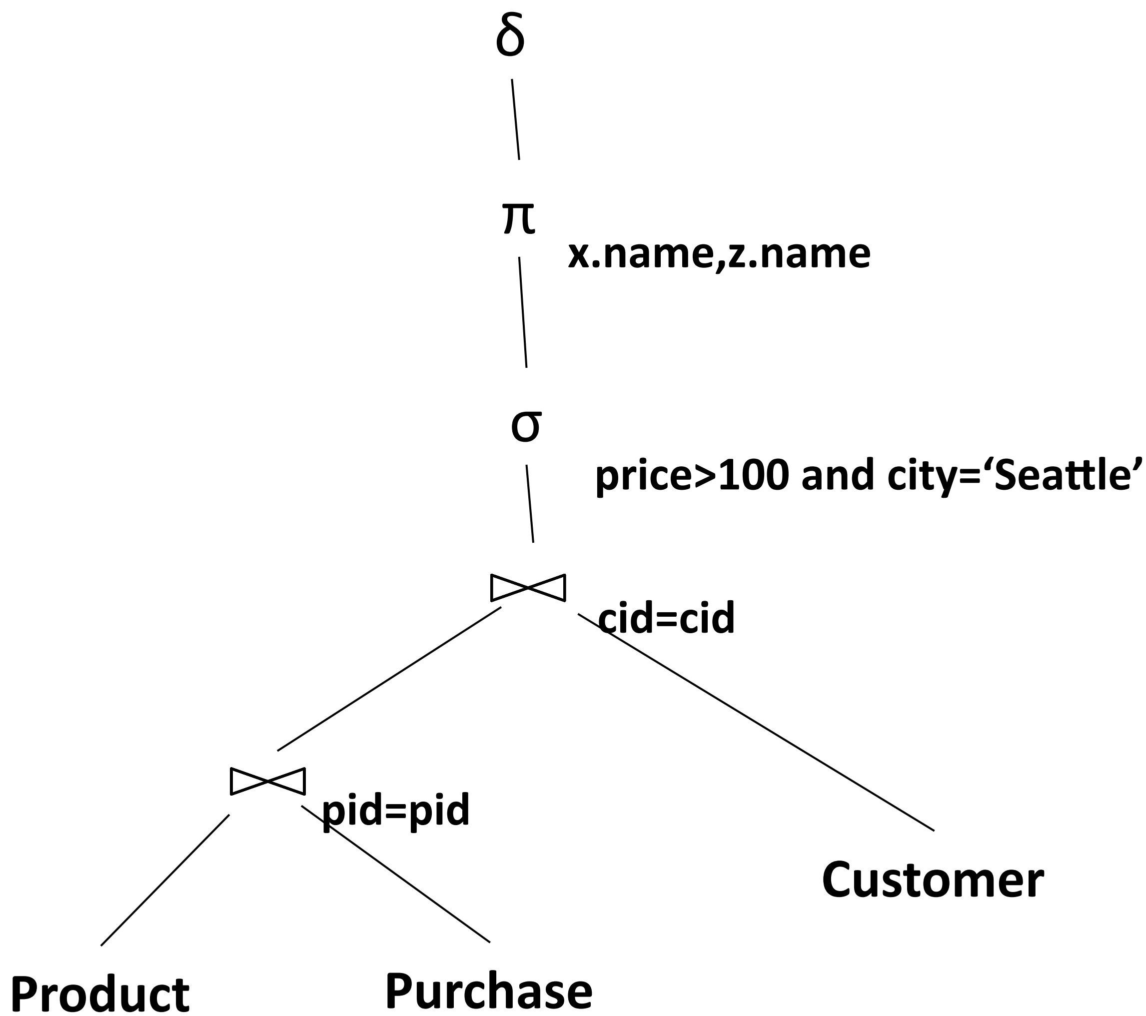
From SQL to RA

Product(<u>pid</u>, name, price)
Purchase(<u>pid, cid</u>, store)
Customer(<u>cid</u>, name, city)

```
SELECT DISTINCT x.name, z.name
FROM Product x, Purchase y, Customer z
WHERE x.pid = y.pid and y.cid = z.cid and
            x.price > 100 and z.city = 'Seattle'
```


x

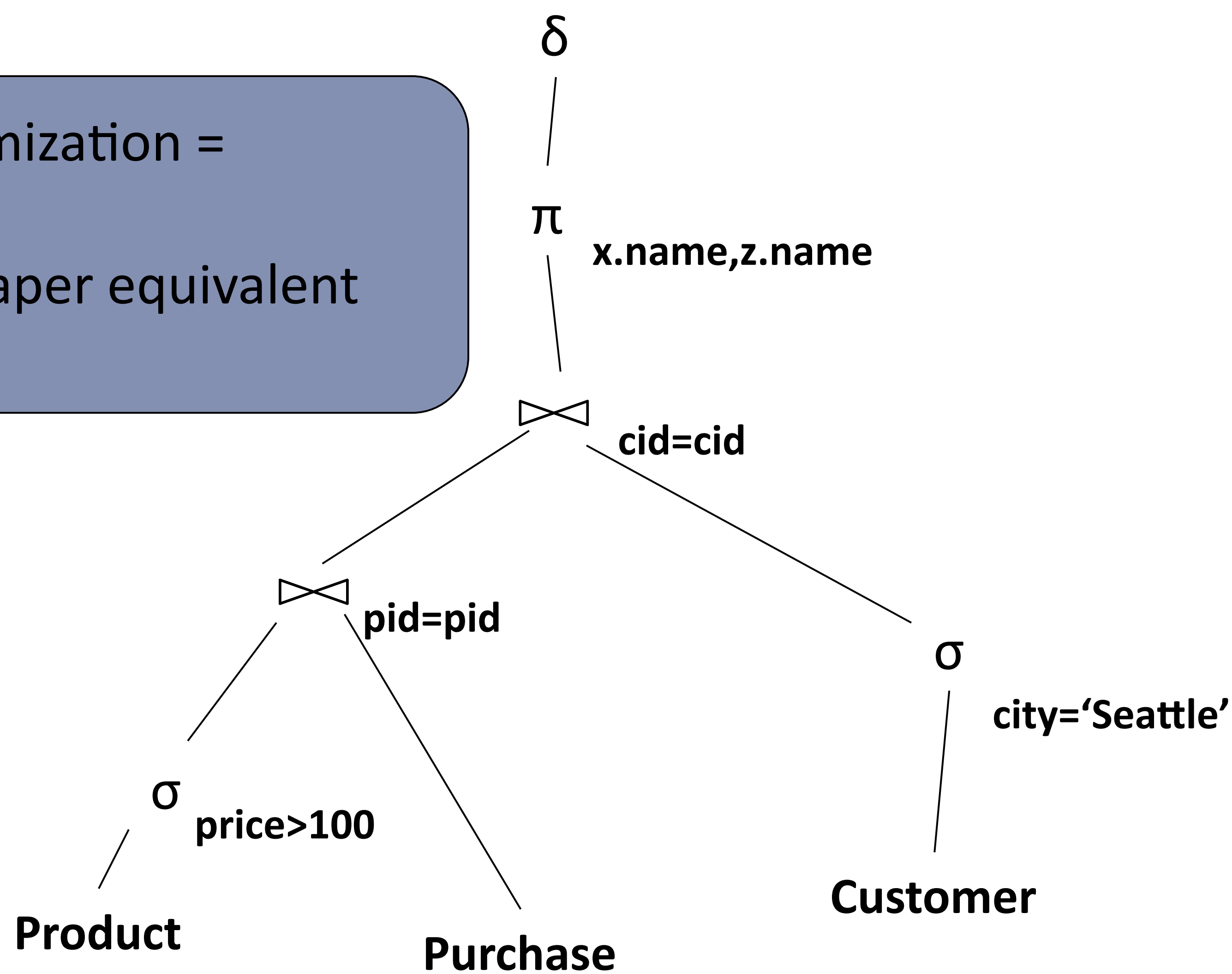
From SQL to RA



An Equivalent Expression

Query optimization =

finding cheaper equivalent expressions



Operators on Bags

- Duplicate elimination δ
- Grouping γ
- Sorting τ

Logical Query Plan

```
SELECT city, count(*)
FROM sales
GROUP BY city
HAVING sum(price) > 100
```

T1, T2, T3 = temporary tables

$\pi_{\text{city, c}}$ T3(city, c)

|

T2(city,p,c)

$\sigma_{p > 100}$

|

T1(city,p,c)

$\gamma_{\text{city, sum(price)} \to \text{p, count(*)} \to \text{c}}$

|

sales(product, city, price)