How cryptography is used to secure web services

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Transfer of Confidential Data

You (client) — Merchant (server)

I want to make a purchase. →

← What is your Credit Card Number?

My Credit Card is 6543 2345 6789 8765. →

Transfer of Confidential Data

- But the Internet provides no privacy.
- Is there any way to protect my data from prying eyes at intermediate nodes?

Symmetric Encryption

- If the user has a pre-existing relationship with the merchant, they may have a shared secret key K – known only to the two parties.
- User encrypts private data with key K.
- Merchant decrypts data with key K.

Asymmetric Encryption

- What if the user and merchant have no prior relationship?
- Asymmetric encryption allows me to encrypt a message for a recipient without knowledge of the recipient’s decryption key.

The Fundamental Equation

$$Z=Y^X \bmod N$$

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When Z is unknown, it can be efficiently computed.

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When X is unknown, the problem is known as the *discrete logarithm* and is generally believed to be hard to solve.

The Fundamental Equation

$$Z=Y^X \bmod N$$

When Y is unknown, the problem is known as *discrete root finding* and is generally believed to be hard to solve … without the factorization of N.

The Fundamental Equation

$$Z=Y^X \bmod N$$

The problem is not well-studied for the case when N is unknown.

How to compute Y^X mod N

Compute Y^X and then reduce mod N.

- If X, Y, and N each are 1,000-bit integers, Y^X consists of $\sim 2^{1010}$ bits.
- Since there are roughly 2^{250} particles in the universe, storage is a problem.

How to compute Y^X mod N

- Repeatedly multiplying by Y (followed each time by a reduction modulo N) X times solves the storage problem.
- However, we would need to perform $\sim 2^{900}$ 32-bit multiplications per second to complete the computation before the sun burns out.

How to compute $Y^X \bmod N$

Multiplication by Repeated Doubling

To compute $X \cdot Y$,

compute $Y, 2Y, 4Y, 8Y, 16Y, \ldots$

and sum up those values dictated by the binary representation of X.

Example: $26Y = 2Y + 8Y + 16Y$.

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How to compute $Y^X \bmod N$

Exponentiation by Repeated Squaring

To compute Y^X,

compute $Y, Y^2, Y^4, Y^8, Y^{16}, \ldots$

and multiply those values dictated by the binary representation of X.

Example: $Y^{26} = Y^2 \cdot Y^8 \cdot Y^{16}$.

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How to compute $Y^X \bmod N$

We can now perform a 1,000-bit modular exponentiation using ~1,500 1,000-bit modular multiplications.

- 1,000 squarings: $y, y^2, y^4, \ldots, y^{2^{1000}}$
- ~500 "ordinary" multiplications

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The Fundamental Equation

$$Z = Y^X \bmod N$$

When Y is unknown, the problem is known as *discrete root finding* and is generally believed to be hard to solve … without the factorization of N.

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RSA Encryption/Decryption

- Select two large primes p and q.
- Publish the product $N = pq$.
- The exponent X is typically fixed at 65537.

- Encrypt message Y as $E(Y) = Y^X \bmod N$.
- Decrypt ciphertext Z as $D(Z) = Z^{1/X} \bmod N$.

- Note $D(E(Y)) = (Y^X)^{1/X} \bmod N = Y$.

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RSA Signatures and Verification

- Not only is $D(E(Y)) = (Y^X)^{1/X} \bmod N = Y$, but also $E(D(Y)) = (Y^{1/X})^X \bmod N = Y$.

- To form a signature of message Y, create $S = D(Y) = Y^{1/X} \bmod N$.

- To verify the signature, check that $E(S) = S^X \bmod N$ matches Y.

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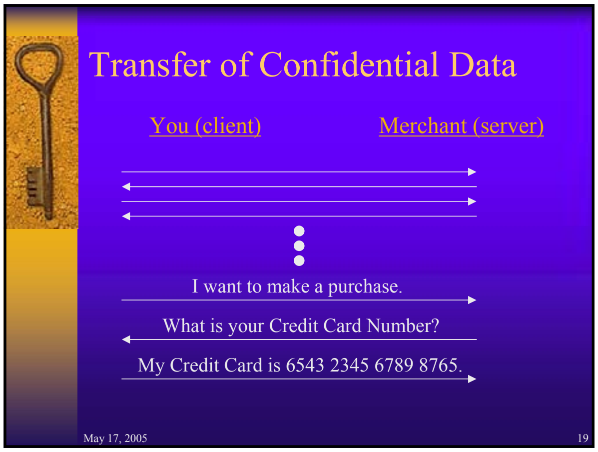
Transfer of Confidential Data
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Merchant (server)
I want to make a purchase.
What is your Credit Card Number?
My Credit Card is 6543 2345 6789 8765.
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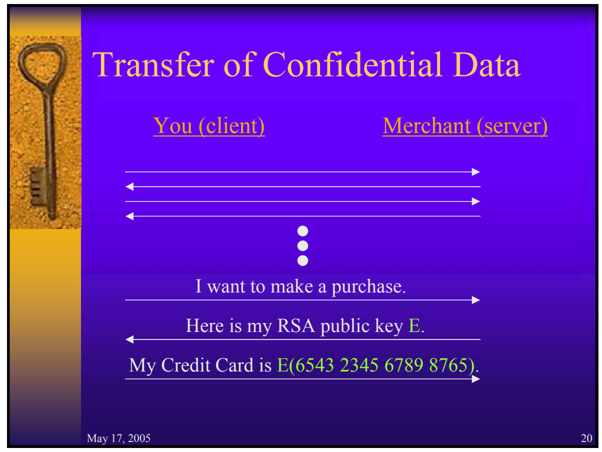
Transfer of Confidential Data
You (client)
Merchant (server)
I want to make a purchase.
Here is my RSA public key E.
My Credit Card is E(6543 2345 6789 8765).
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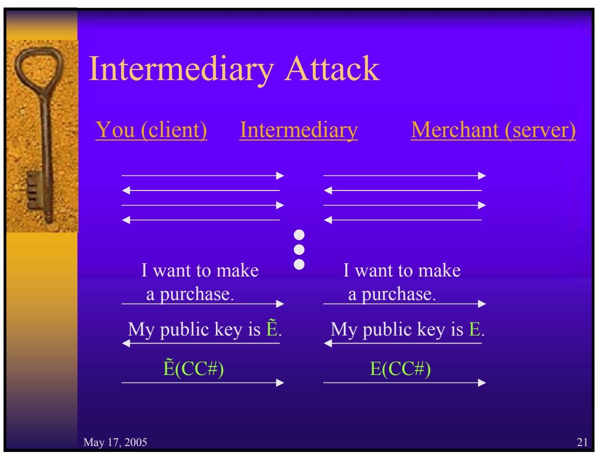
Intermediary Attack
You (client)
Intermediary
Merchant (server)
I want to make a purchase.
My public key is Ẽ.
Ẽ(CC#)
I want to make a purchase.
My public key is E.
E(CC#)
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Digital Certificates
"Alice's public modulus is N_A = 331490324840…"
-- signed …
someone you trust.
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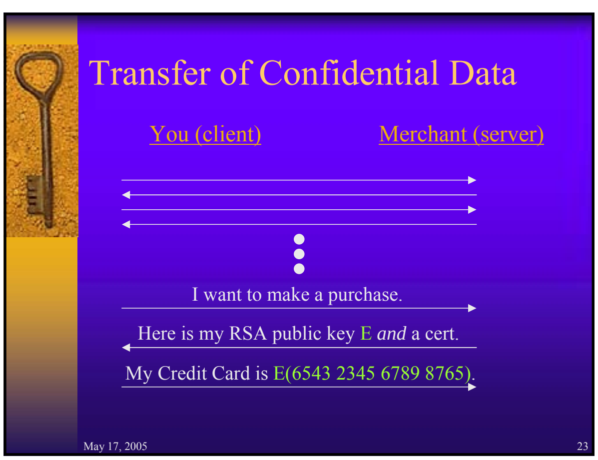
Transfer of Confidential Data
You (client)
Merchant (server)
I want to make a purchase.
Here is my RSA public key E *and* a cert.
My Credit Card is E(6543 2345 6789 8765).
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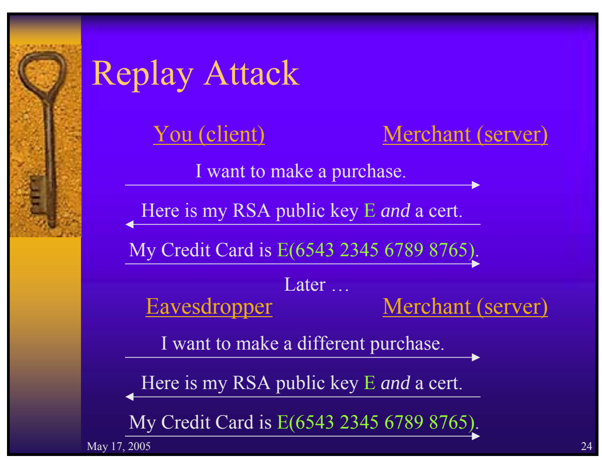
Replay Attack
You (client)
Merchant (server)
I want to make a purchase.
Here is my RSA public key E *and* a cert.
My Credit Card is E(6543 2345 6789 8765).
Later …
Eavesdropper
Merchant (server)
I want to make a different purchase.
Here is my RSA public key E *and* a cert.
My Credit Card is E(6543 2345 6789 8765).
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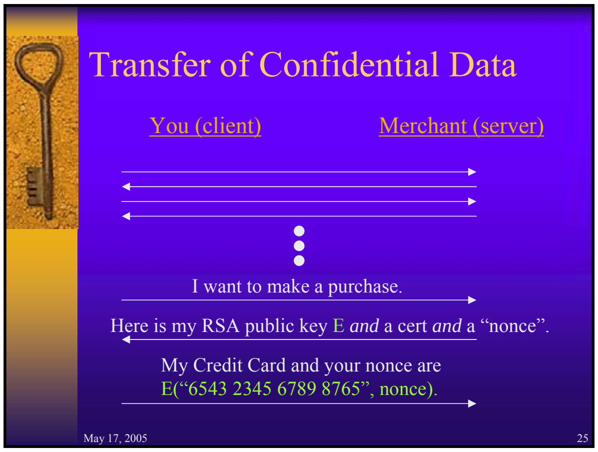
Transfer of Confidential Data
You (client)
Merchant (server)
I want to make a purchase.
Here is my RSA public key E and a cert and a "nonce".
My Credit Card and your nonce are
E("6543 2345 6789 8765", nonce).
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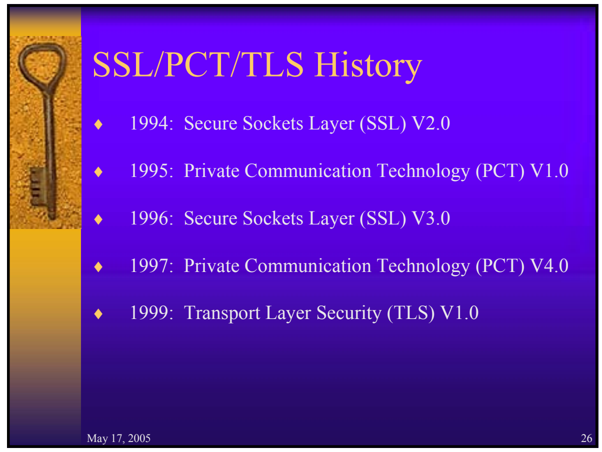
SSL/PCT/TLS History
1994: Secure Sockets Layer (SSL) V2.0
1995: Private Communication Technology (PCT) V1.0
1996: Secure Sockets Layer (SSL) V3.0
1997: Private Communication Technology (PCT) V4.0
1999: Transport Layer Security (TLS) V1.0
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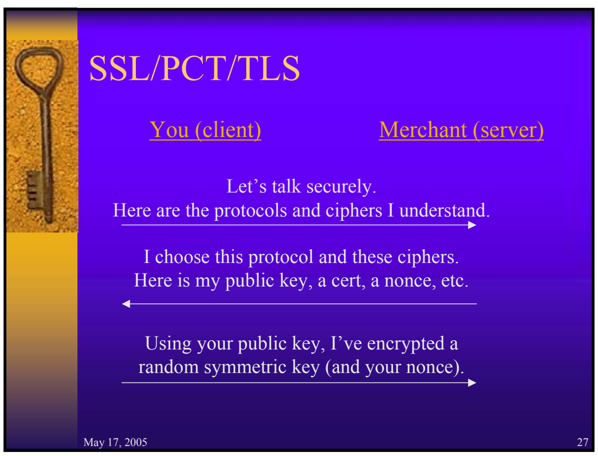
SSL/PCT/TLS
You (client)
Merchant (server)
Let's talk securely.
Here are the protocols and ciphers I understand.
I choose this protocol and these ciphers.
Here is my public key, a cert, a nonce, etc.
Using your public key, I've encrypted a
random symmetric key (and your nonce).
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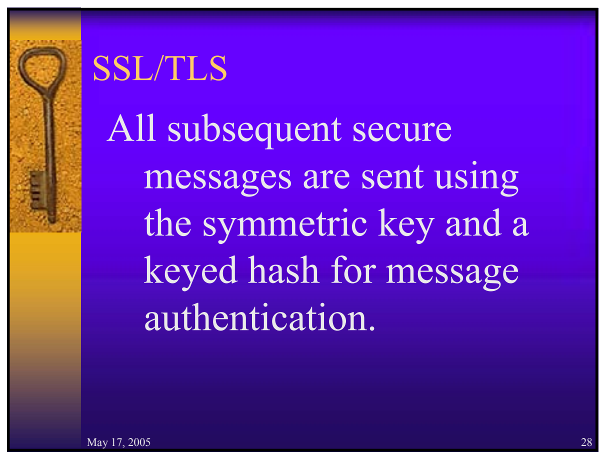
SSL/TLS
All subsequent secure messages are sent using the symmetric key and a keyed hash for message authentication.
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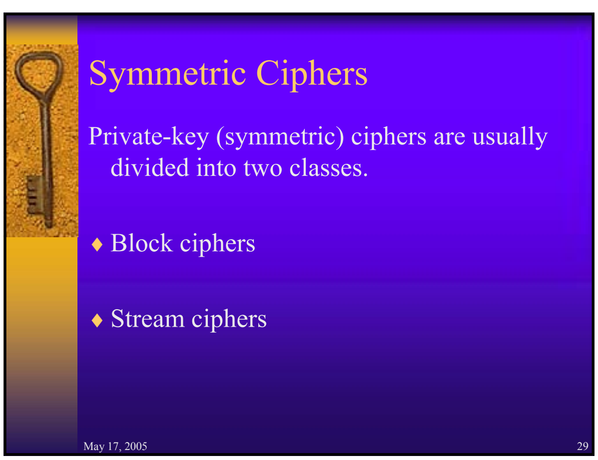
Symmetric Ciphers
Private-key (symmetric) ciphers are usually divided into two classes.
Block ciphers
Stream ciphers
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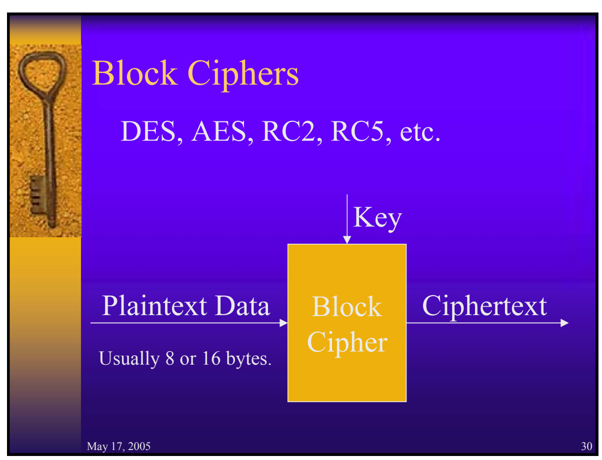
Block Ciphers
DES, AES, RC2, RC5, etc.
Key
Plaintext Data
Block Cipher
Ciphertext
Usually 8 or 16 bytes.
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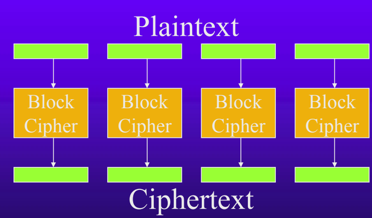
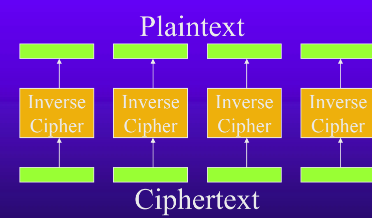
Block Cipher Modes

Electronic Code Book (ECB) Encryption:



Block Cipher Modes

Electronic Code Book (ECB) Decryption:



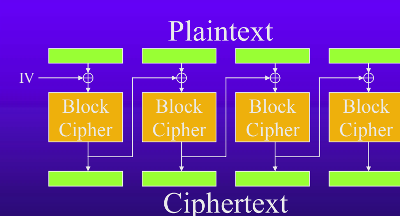
Block Cipher Integrity

With ECB mode, identical blocks will have identical encryptions.

This can enable replay attacks as well as re-orderings of data. Even a passive observer may obtain statistical data.

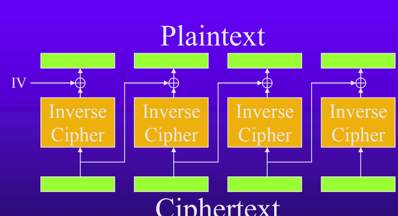
Block Cipher Modes

Cipher Block Chaining (CBC) Encryption:



Block Cipher Modes

Cipher Block Chaining (CBC) Decryption:

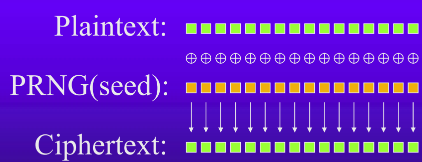


Stream Ciphers

RC4, SEAL, etc.

- Use the key as a seed to a pseudo-random number-generator.
- Take the stream of output bits from the PRNG and XOR it with the plaintext to form the ciphertext.

Stream Cipher Encryption



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Stream Cipher Integrity

- It is easy for an adversary (even one who can't decrypt the ciphertext) to alter the plaintext in a known way.

Bob to Bob's Bank:

Please transfer $0,000,002.00 to the account of my good friend Alice.

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Stream Cipher Integrity

- It is easy for an adversary (even one who can't decrypt the ciphertext) to alter the plaintext in a known way.

Bob to Bob's Bank:

Please transfer $1,000,002.00 to the account of my good friend Alice.

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One-Way Hash Functions

The idea of a *check sum* is great, but it is designed to prevent accidental changes in a message.

For cryptographic integrity, we need an integrity check that is resilient against a smart and determined adversary.

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One-Way Hash Functions

MD4, MD5, SHA-1, SHA-256, etc.

Generally, a *one-way hash function* is a function $H : \{0,1\}^* \rightarrow \{0,1\}^k$ (typically k is 128 or 160) such that given an input value x, one cannot find a value $x' \neq x$ such $H(x) = H(x')$.

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One-Way Hash Functions

There are many measures for one-way hashes.

- Non-invertability: given y, it's difficult to find any x such that $H(x) = y$.
- Collision-intractability: one cannot find a pair of values $x' \neq x$ such that $H(x) = H(x')$.

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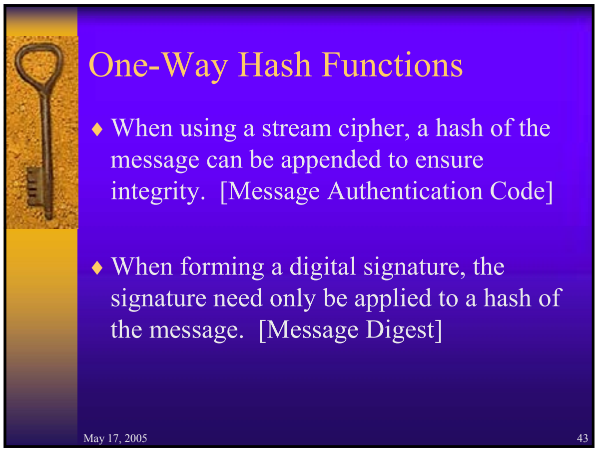
One-Way Hash Functions
♦ When using a stream cipher, a hash of the message can be appended to ensure integrity. [Message Authentication Code]
♦ When forming a digital signature, the signature need only be applied to a hash of the message. [Message Digest]
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