Motion Estimation
Readings: Ch 9: 9.1-9.3 plus papers

- change detection
- optical flow analysis
- Lucas-Kanade method with pyramid structure
- Ming Ye’s improved method

Why estimate motion?

We live in a 4-D world

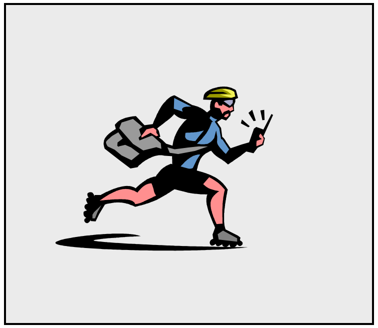
Wide applications

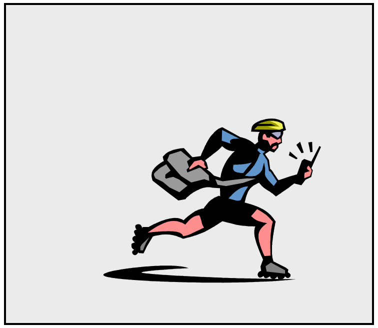
- Object Tracking
- Camera Stabilization
- Image Mosaics
- 3D Shape Reconstruction (SFM)
- Special Effects (Match Move)

Frame from an ARDA Sample Video

Change detection for surveillance

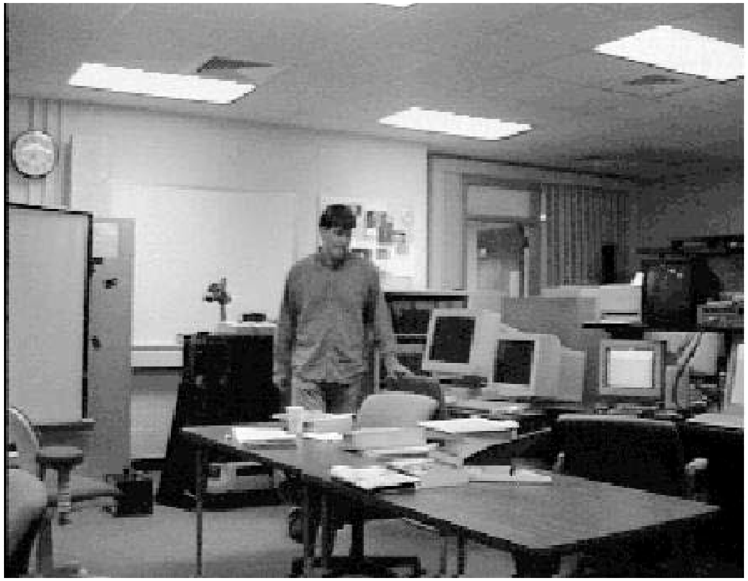
- Video frames: F1, F2, F3, …
- Objects appear, move, disappear
- Background pixels remain the same (simple case)





- How do you detect the moving objects?
- Simple answer: pixelwise subtraction

Example: Person detected entering room



- Pixel changes detected as difference components
- Regions are (1) person, (2) opened door, and (3) computer monitor.
- System can know about the door and monitor. Only the person region is “unexpected”.

Change Detection via Image Subtraction

for each pixel [r,c]
  if (|I1[r,c] - I2[r,c]| > threshold) then Iout[r,c] = 1 else Iout[r,c] = 0

Perform connected components on Iout.

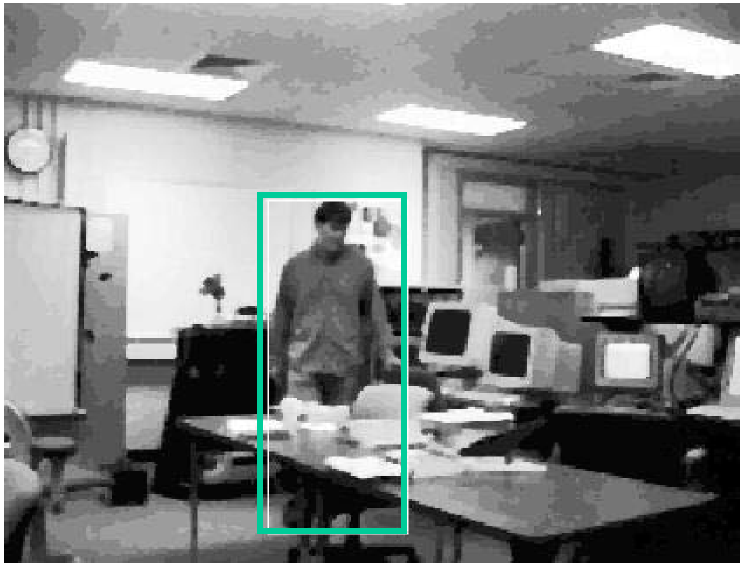
Remove small regions.

Perform a closing with a small disk for merging close neighbors.

Compute and return the bounding boxes B of each remaining region.

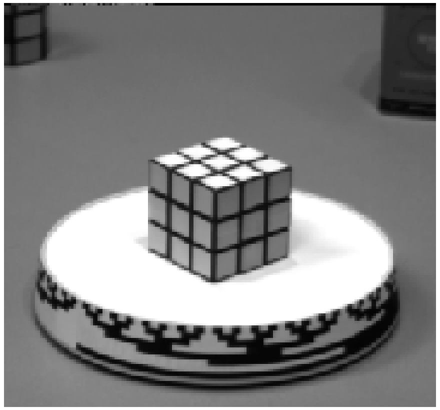
What assumption does this make about the changes?

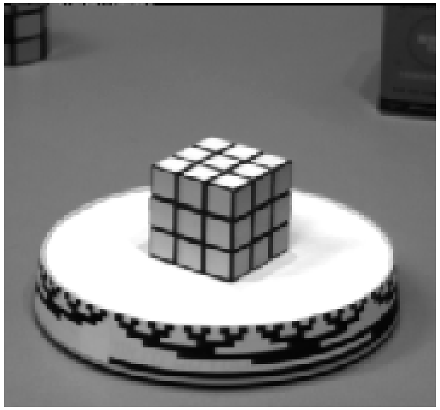
Change analysis

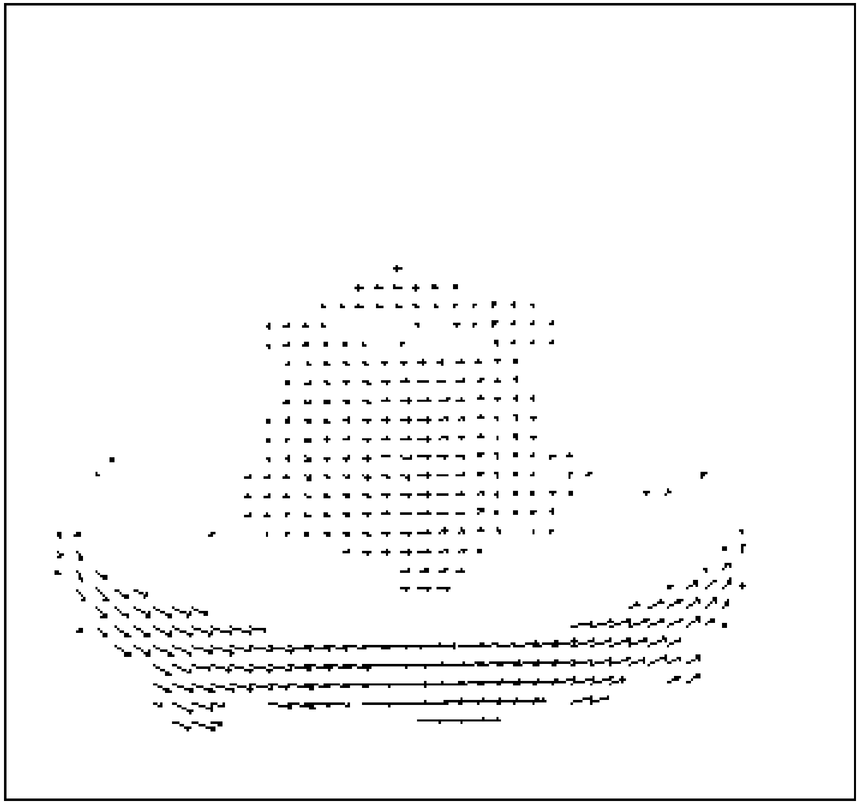


Known regions are ignored and system attends to the unexpected region of change. Region has bounding box similar to that of a person. System might then zoom in on "head" area and attempt face recognition.

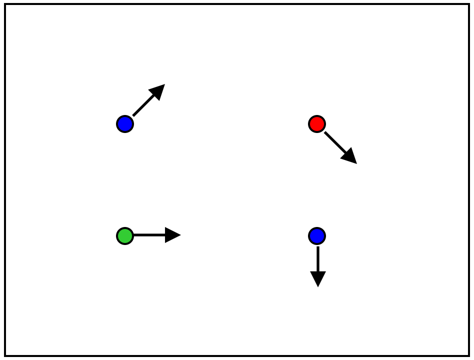
Optical flow



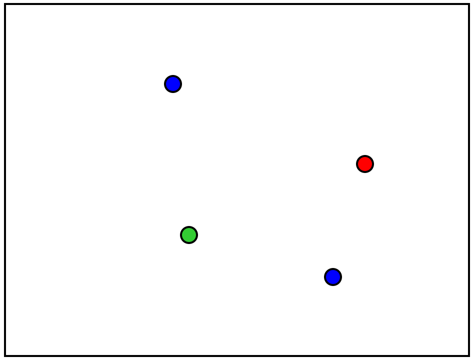




Problem definition: optical flow



$H(x, y)$



$I(x, y)$

How to estimate pixel motion from image H to image I?

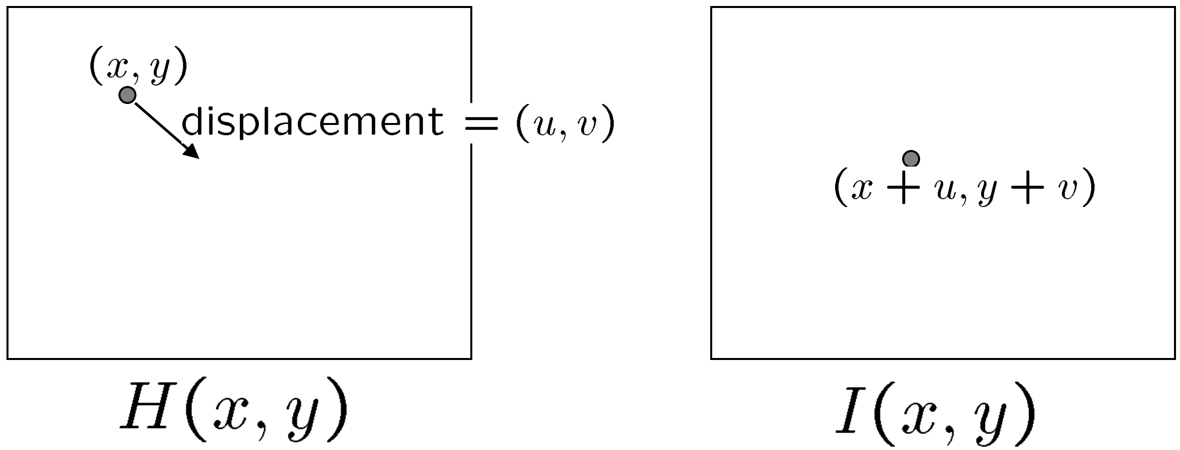
- Solve pixel correspondence problem
  - given a pixel in H, look for nearby pixels of the same color in I

Key assumptions

- **color constancy:** a point in H looks the same in I
  - For grayscale images, this is **brightness constancy**
- **small motion**: points do not move very far

This is called the **optical flow** problem

Optical flow constraints (grayscale images)



Let's look at these constraints more closely

- brightness constancy: Q: what's the equation?

$$H(x, y) = I(x+u, y+v)$$

- small motion: (u and v are less than 1 pixel)
  - suppose we take the Taylor series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

$$\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

Optical flow equation

Combining these two equations

shorthand: $I_x = \frac{\partial I}{\partial x}$

The x-component of the gradient vector.

$$
\begin{aligned}
0 &= I(x+u, y+v) - H(x,y) \\
&\approx I(x,y) + I_x u + I_y v - H(x,y) \\
&\approx (I(x,y) - H(x,y)) + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v \\
&\approx I_t + \nabla I \cdot [u \;\; v]
\end{aligned}
$$

What is I_t ? The time derivative of the image at (x,y)

How do we calculate it?

Optical flow equation

$$0 = I_t + \nabla I \cdot [u \;\; v]$$

Q: how many unknowns and equations per pixel?

1 equation, but 2 unknowns (u and v)

Intuitively, what does this constraint mean?

- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown

Lukas-Kanade flow

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - » If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\underset{\substack{A \\ 25\times 2}}{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}} \underset{\substack{d \\ 2\times 1}}{\begin{bmatrix} u \\ v \end{bmatrix}} = - \underset{\substack{b \\ 25\times 1}}{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}$$

RGB version

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - If we use a 5x5 window, that gives us 25*3 equations per pixel!

$$0 = I_t(\mathbf{p_i})[0,1,2] + \nabla I(\mathbf{p_i})[0,1,2] \cdot [u \;\; v]$$

$$\underset{\substack{A \\ 75\times 2}}{\begin{bmatrix} I_x(\mathbf{p_1})[0] & I_y(\mathbf{p_1})[0] \\ I_x(\mathbf{p_1})[1] & I_y(\mathbf{p_1})[1] \\ I_x(\mathbf{p_1})[2] & I_y(\mathbf{p_1})[2] \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}})[0] & I_y(\mathbf{p_{25}})[0] \\ I_x(\mathbf{p_{25}})[1] & I_y(\mathbf{p_{25}})[1] \\ I_x(\mathbf{p_{25}})[2] & I_y(\mathbf{p_{25}})[2] \end{bmatrix}} \underset{\substack{d \\ 2\times 1}}{\begin{bmatrix} u \\ v \end{bmatrix}} = - \underset{\substack{b \\ 75\times 1}}{\begin{bmatrix} I_t(\mathbf{p_1})[0] \\ I_t(\mathbf{p_1})[1] \\ I_t(\mathbf{p_1})[2] \\ \vdots \\ I_t(\mathbf{p_{25}})[0] \\ I_t(\mathbf{p_{25}})[1] \\ I_t(\mathbf{p_{25}})[2] \end{bmatrix}}$$

Lukas-Kanade flow

Prob: we have more equations than unknowns

$$\underset{25\times2}{A}\ \underset{2\times1}{d} = \underset{25\times1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

- minimum least squares solution given by solution (in d) of:

$$\underset{2\times2}{(A^T A)}\ \underset{2\times1}{d} = \underset{2\times1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- The summations are over all pixels in the K x K window
- This technique was first proposed by Lukas & Kanade for stereo matching (1981)

Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

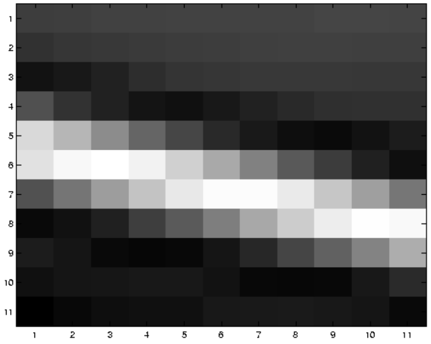
$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

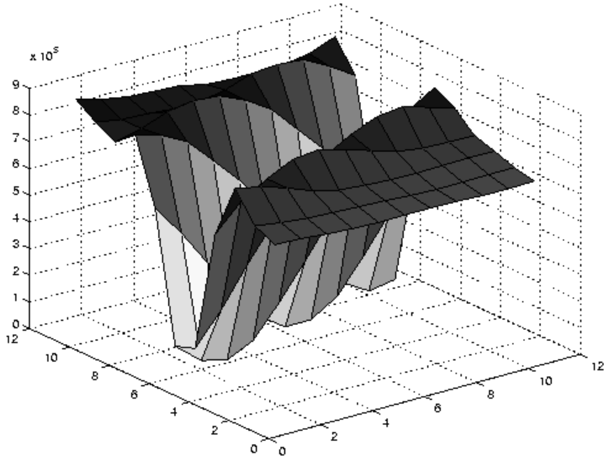
$$A^T A \qquad\qquad\qquad\qquad A^T b$$

When is This Solvable?

- **A^TA** should be invertible
- **A^TA** should not be too small due to noise
  – eigenvalues λ_1 and λ_2 of **A^TA** should not be too small
- **A^TA** should be well-conditioned
  – λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

Edges cause problems



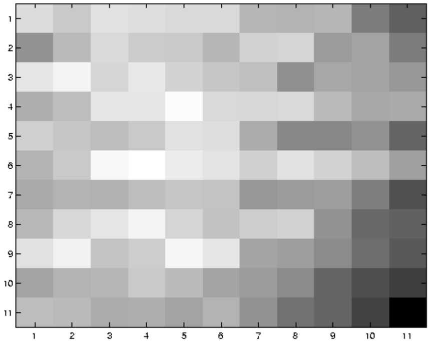


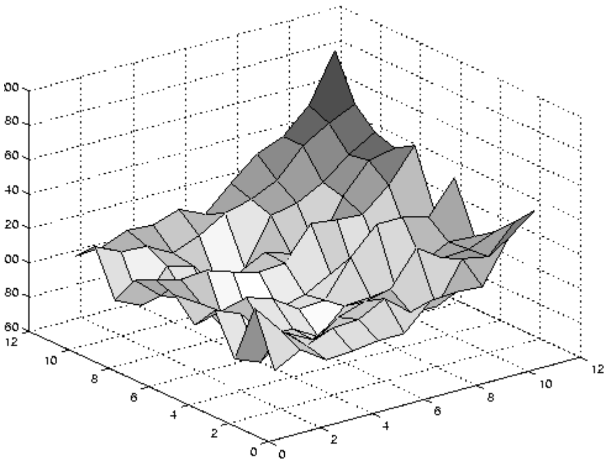
$$\sum \nabla I (\nabla I)^T$$

– large gradients, all the same

– large λ_1, small λ_2

Low texture regions don't work



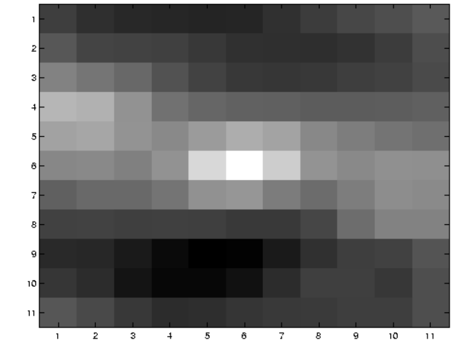


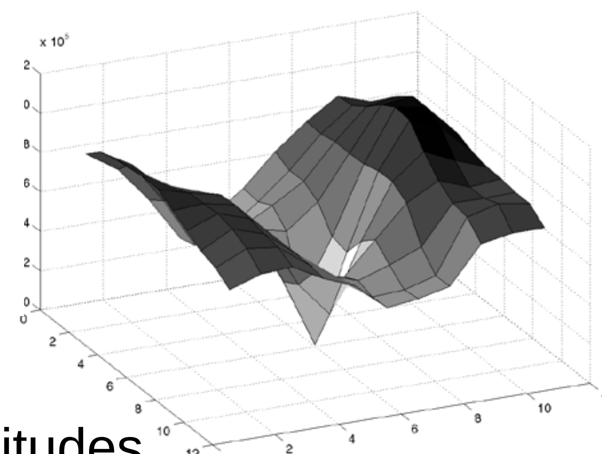
$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

High textured region work best





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Errors in Lukas-Kanade

What are the potential causes of errors in this procedure?

- Suppose A^TA is easily invertible
- Suppose there is not much noise in the image

When our assumptions are violated

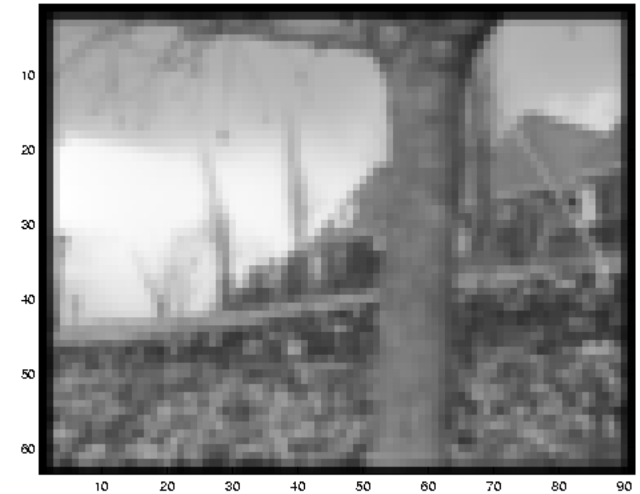
- Brightness constancy is **not** satisfied
- The motion is **not** small
- A point does **not** move like its neighbors
  - window size is too large
  - what is the ideal window size?

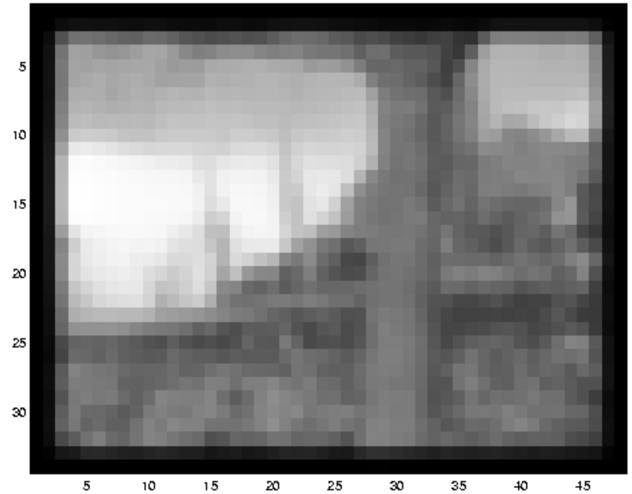
Revisiting the small motion assumption

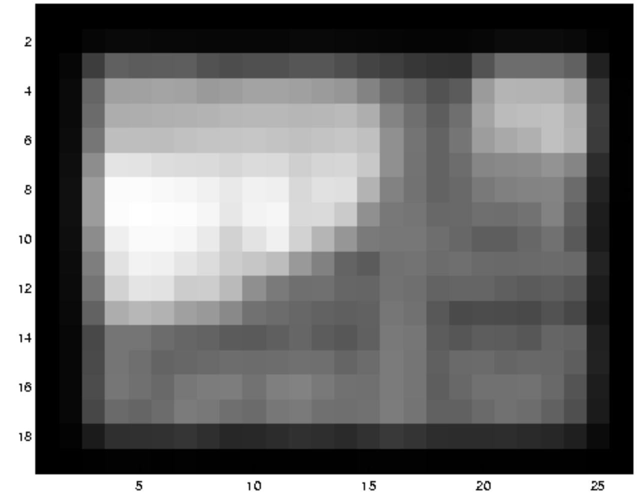
Is this motion small enough?

- Probably not—it's much larger than one pixel (2^{nd} order terms dominate)
- How might we solve this problem?

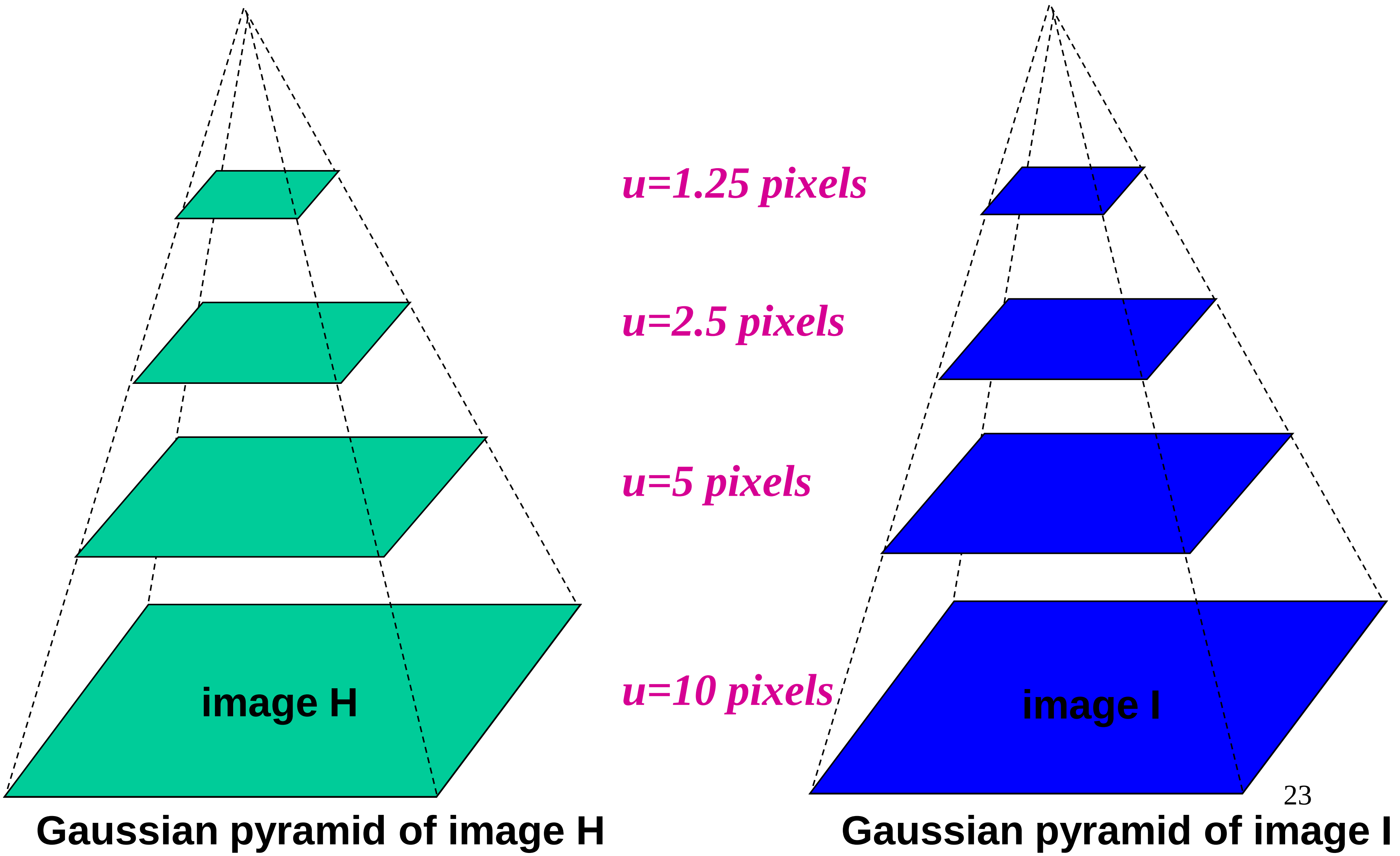
Reduce the resolution!



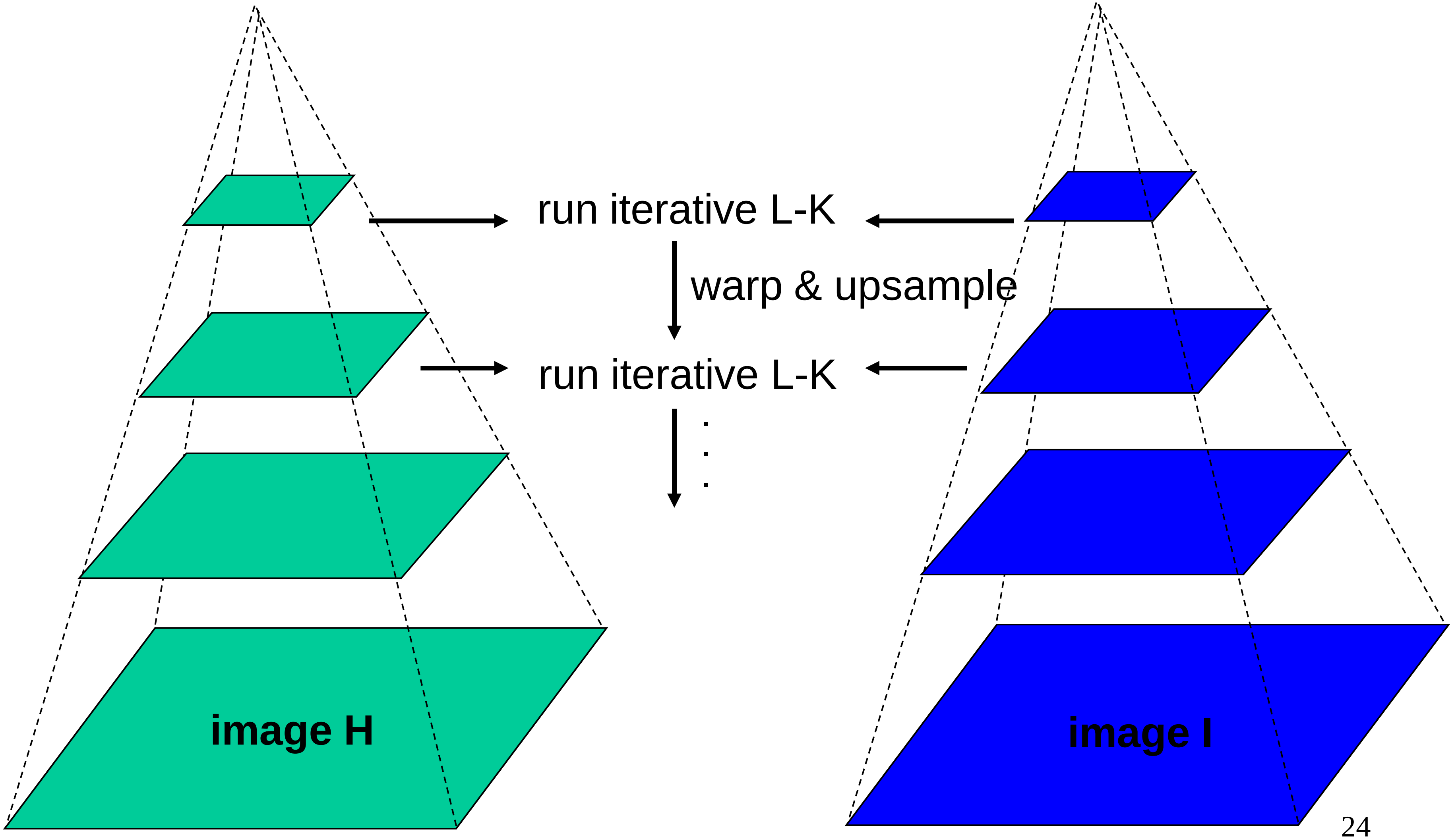




Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation



Gaussian pyramid of image H

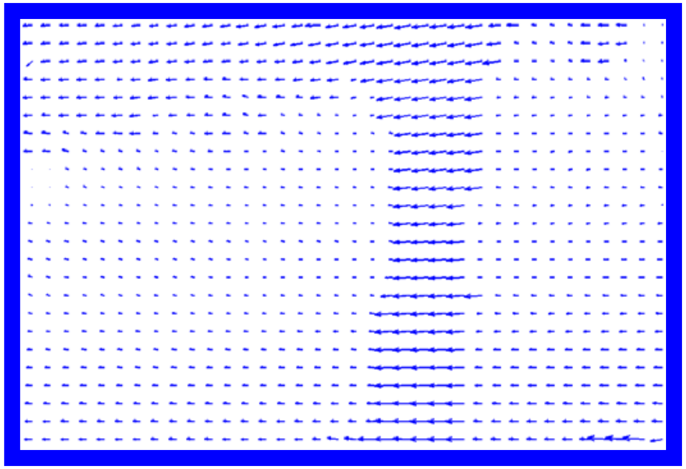
Gaussian pyramid of image I

A Few Details

- Top Level
  - Apply L-K to get a flow field representing the flow from the first frame to the second frame.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K on the new warped image to get a flow field from it to the second frame.
  - Repeat till convergence.
- Next Level
  - Upsample the flow field to the next level as the first guess of the flow at that level.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K and warping till convergence as above.
- Etc.

The Flower Garden Video

What should the optical flow be?



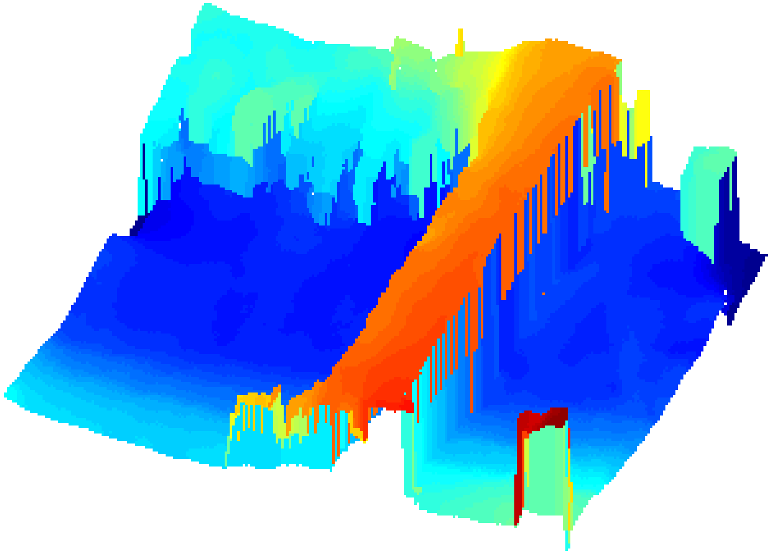
Robust Visual Motion Analysis:
Piecewise-Smooth Optical Flow

Ming Ye
Electrical Engineering
University of Washington

Structure From Motion

Rigid scene + camera translation

Estimated horizontal motion



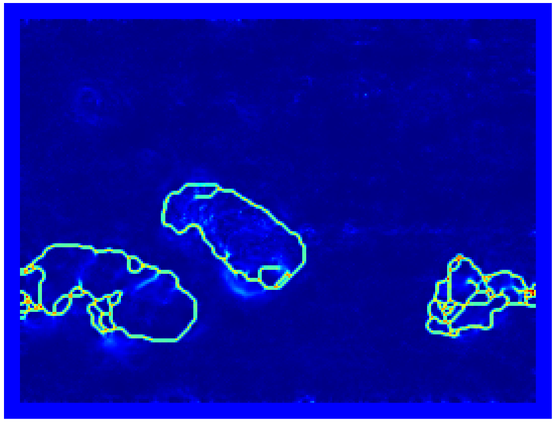
Depth map

Scene Dynamics Understanding

Brighter pixels => larger speeds.

Estimated horizontal motion

- Surveillance
- Event analysis
- Video compression



Motion boundaries are smooth.

Motion smoothness

Target Detection and Tracking

A tiny airplane --- only observable by its distinct motion

Tracking results

Estimating Piecewise-Smooth Optical Flow with Global Matching and Graduated Optimization

Problem Statement:

Assuming only brightness conservation and piecewise-smooth motion, find the optical flow to best describe the intensity change in three frames.

Approach: Matching-Based Global Optimization

- **Step 1. Robust local gradient-based method for high-quality initial flow estimate.**
- **Step 2. Global gradient-based method to improve the flow-field coherence.**
- **Step 3. Global matching that minimizes energy by a greedy approach.**

Global Energy Design

Global energy

$$E = \sum_{\text{all sites s}} E_B(V_s) + E_S(V_s)$$

- V is the optical flow field.
- V_s is the optical flow at pixel (site) s.
- E_B is the brightness conservation error.
- E_S is the flow smoothness error in a neighborhood about pixel s.

Global Energy Design

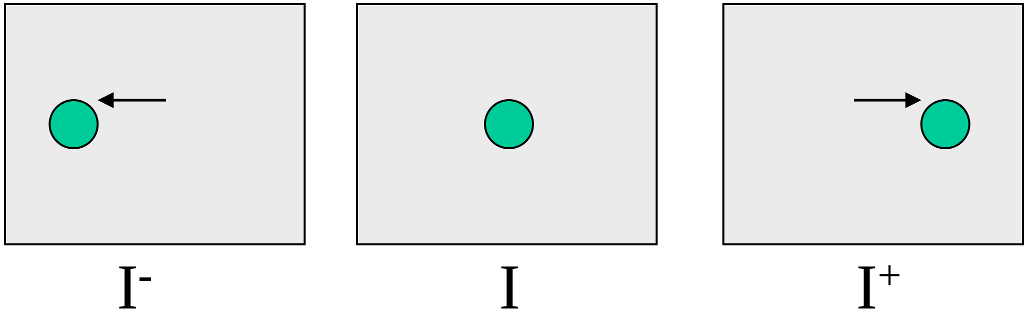
Brightness error

$$E_B(V_s) = \rho(e_W(V_s), \sigma_{B_s})$$

warping error

$$e_W(V_s) = \min(|I^-(V_s) - I_s|, |I^+(V_s) - I_s|)$$

$I^-(V_s)$ is the warped intensity in the previous frame.
$I^+(V_s)$ is the warped intensity in the next frame.



Error function: $\rho(x,\sigma) = \dfrac{x^2}{\sigma^2 + x^2}$ where σ is a scale parameter.

Global Energy Design

Smoothness error

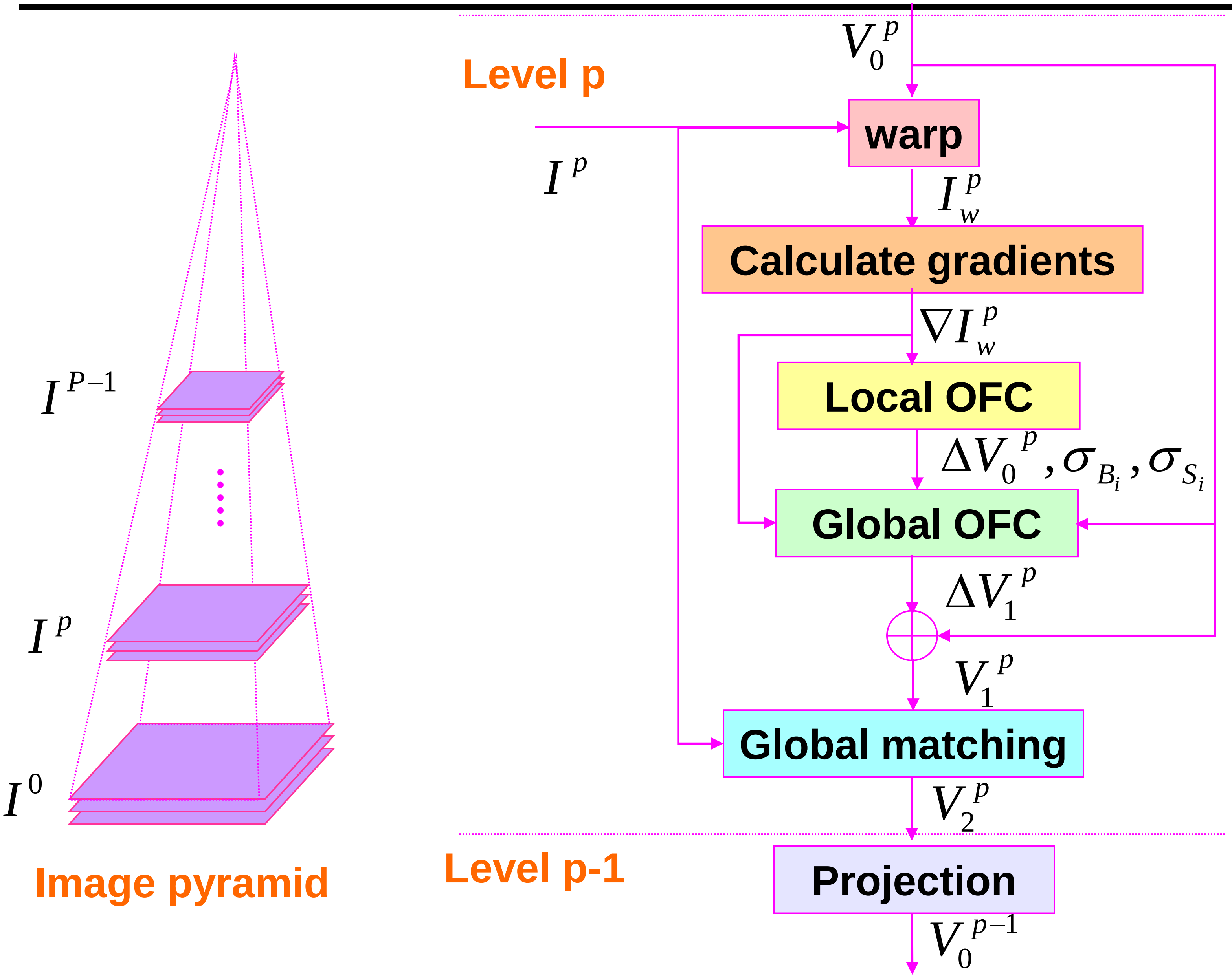
$$E_S(V_i) = \frac{1}{8} \sum_{n \in N_s^8} \rho(|V_s - V_n|, \sigma_{S_s})$$

Smoothness error is computed in a neighborhood around pixel s.

$\mathrm{V_{nw}}$	$\mathrm{V_n}$	$\mathrm{V_{ne}}$
$\mathrm{V_w}$	$\mathbf{V_s}$	$\mathrm{V_e}$
$\mathrm{V_{sw}}$	$\mathrm{V_s}$	$\mathrm{V_{se}}$

Error function: $\rho(x, \sigma) = \frac{x^2}{\sigma^2 + x^2}$

Overall Algorithm



Advantages

Best of Everything

- Local OFC
  - High-quality initial flow estimates
  - Robust local scale estimates
- Global OFC
  - Improve flow smoothness
- Global Matching
  - The optimal formulation
  - Correct errors caused by poor gradient quality and hierarchical process

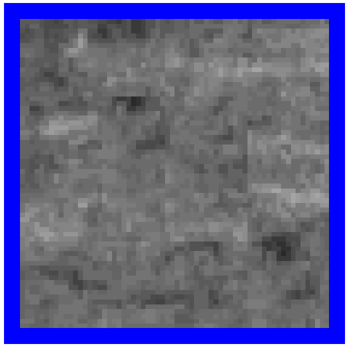
Results: fast convergence, high accuracy, simultaneous motion boundary detection

Experiments

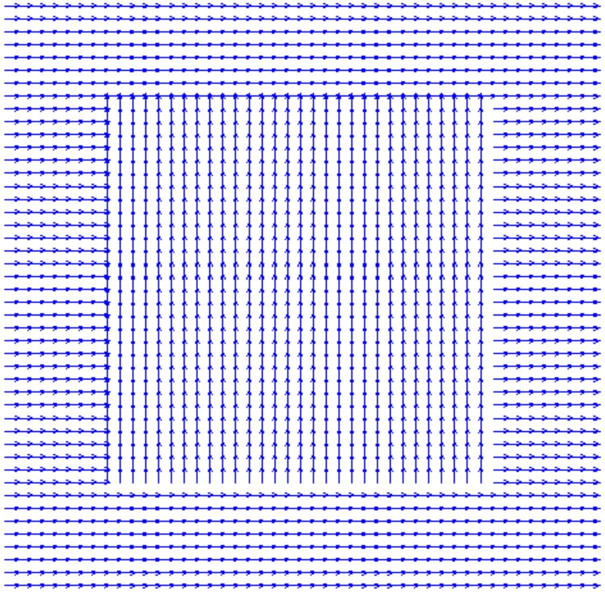
- **Experiments were run on several standard test videos.**
- **Estimates of optical flow were made for the middle frame of every three.**
- **The results were compared with the Black and Anandan algorithm.**

TS: Translating Squares

Homebrew, ideal setting, test performance upper bound

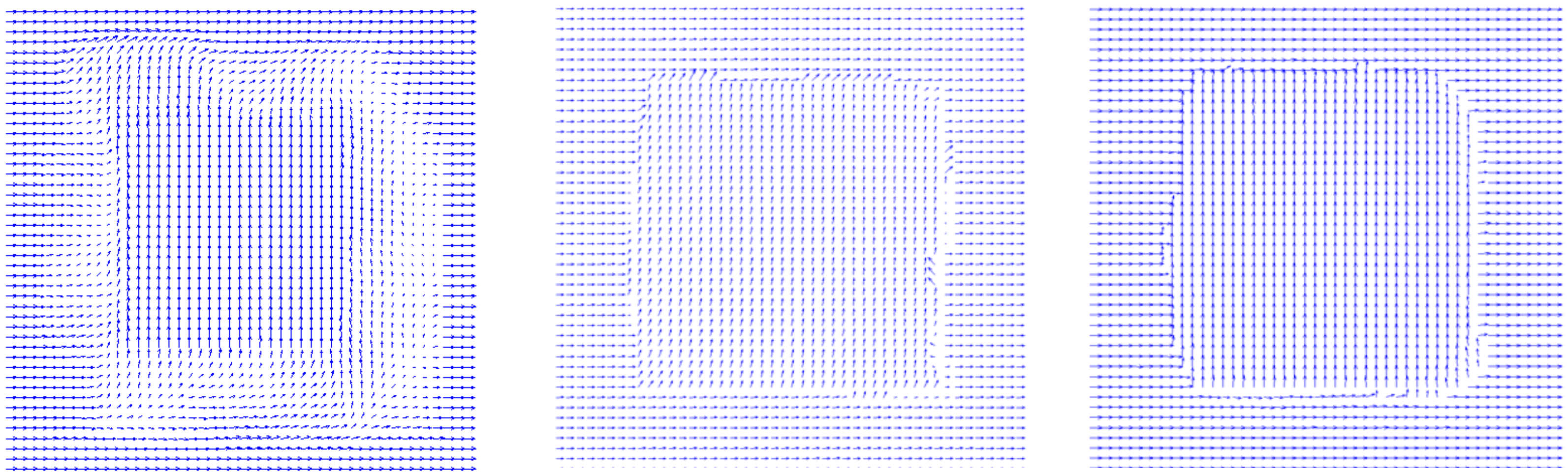


64x64, 1pixel/frame



Groundtruth (cropped),
Our estimate looks the same

TS: Flow Estimate Plots



LS BA S1 (S2 is close)

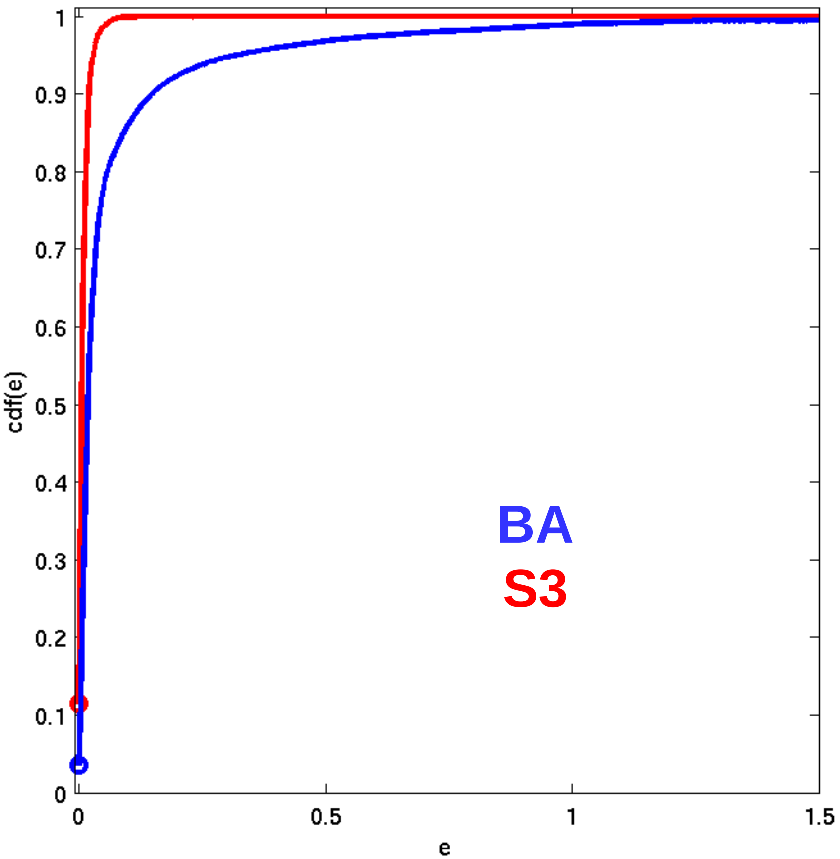
S3 looks the same as the groundtruth.

- S1, S2, S3: results from our Step I, II, III (final)

TT: Translating Tree

150x150 (Barron 94)

	$e_{\angle}(^\circ)$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	2.60	0.128	0.0724
S3	0.248	0.0167	0.00984

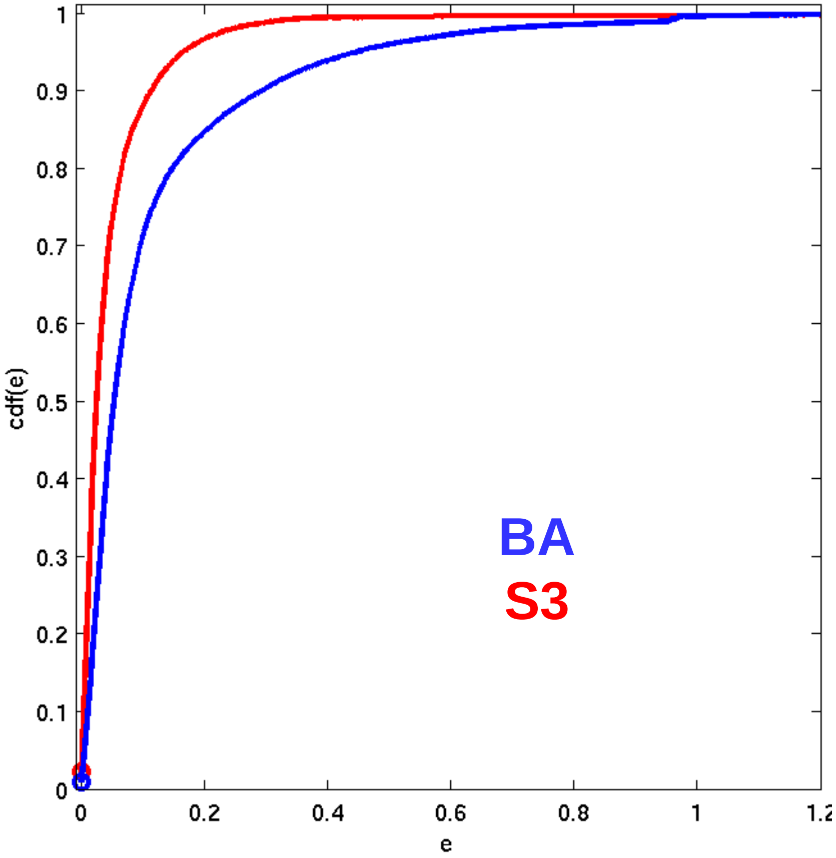


e: error in pixels, cdf: culmulative distribution function for all pixels

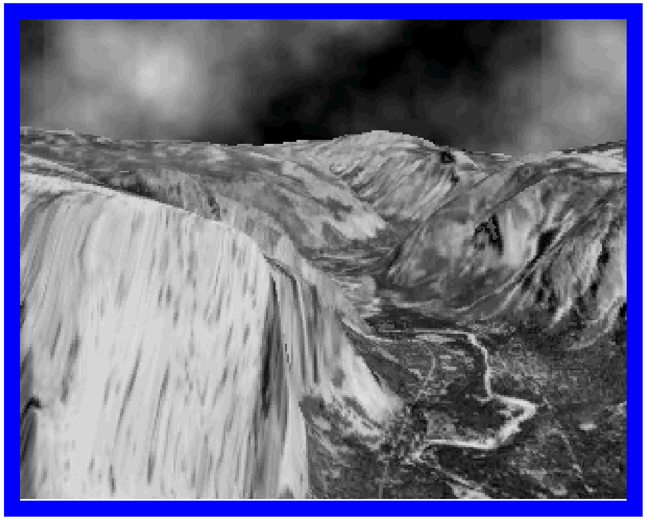
DT: Diverging Tree

150x150 (Barron 94)

	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	6.36	0.182	0.114
S3	2.60	0.0813	0.0507

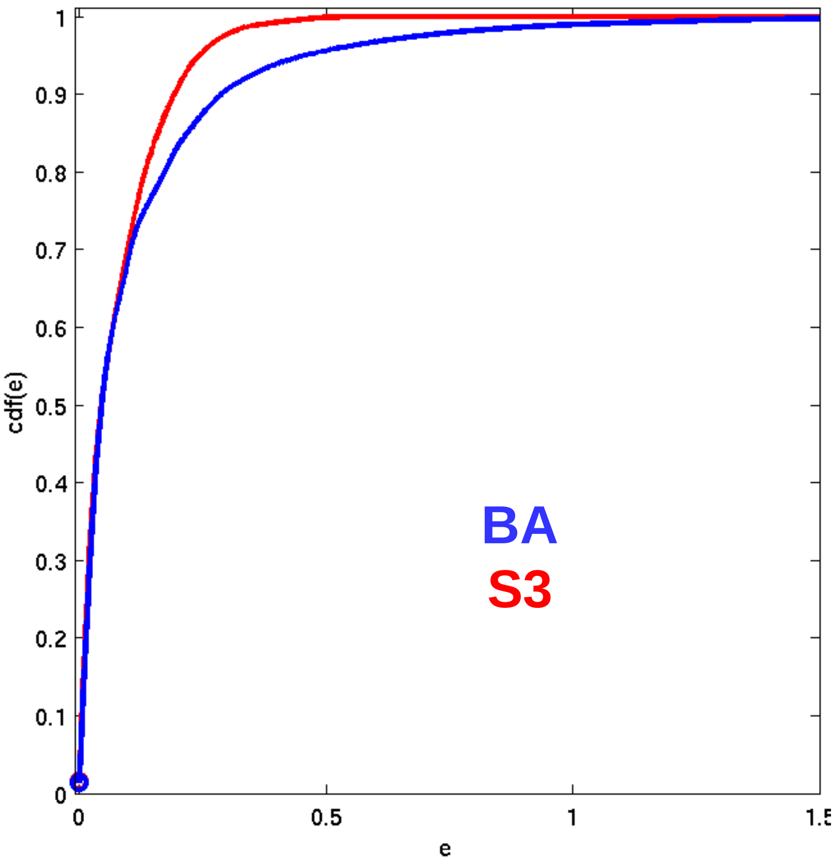


YOS: Yosemite Fly-Through



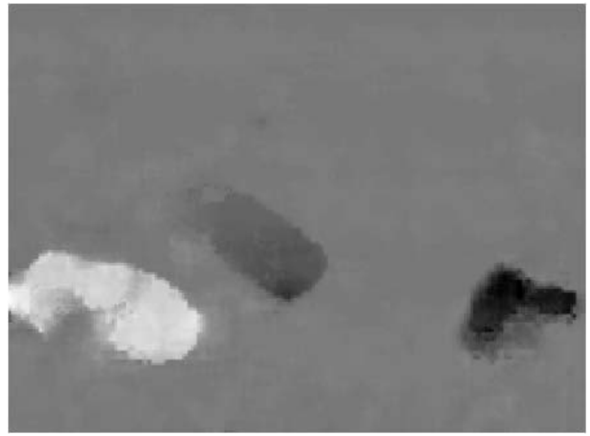
316x252 (Barron, cloud excluded)

	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\overline{e}(\text{pix})$
BA	**2.71**	**0.185**	**0.118**
S3	**1.92**	**0.120**	**0.0776**



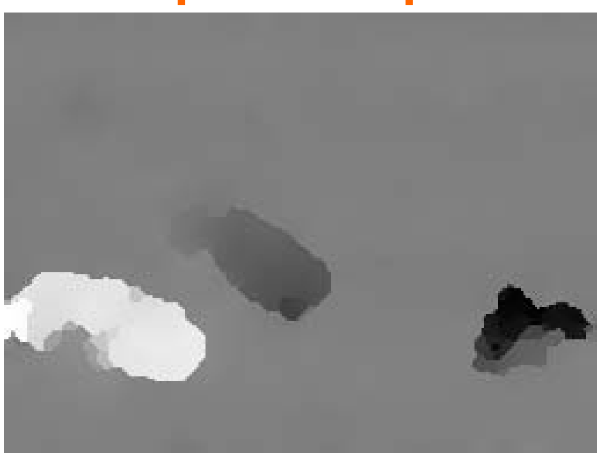
TAXI: Hamburg Taxi

256x190, (Barron 94)
max speed 3.0 pix/frame



LMS

BA



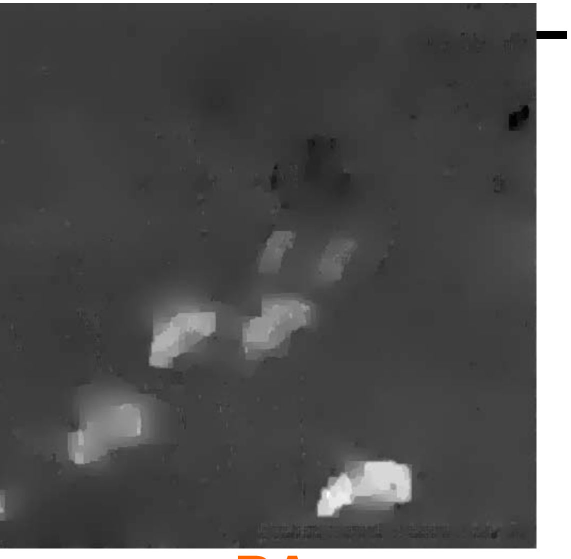
Ours

Error map

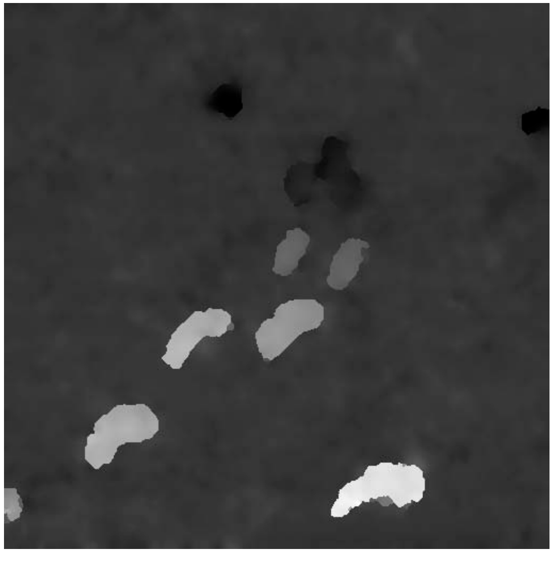
Smoothness error

Traffic

512x512
(Nagel)
max speed:
6.0 pix/frame

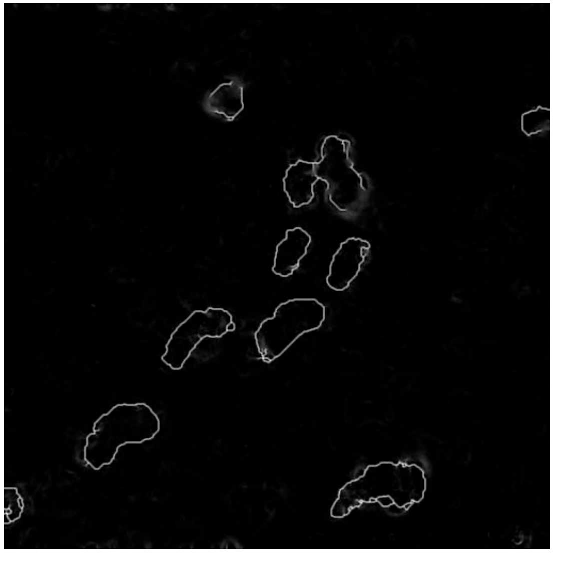


BA



Ours

Error map



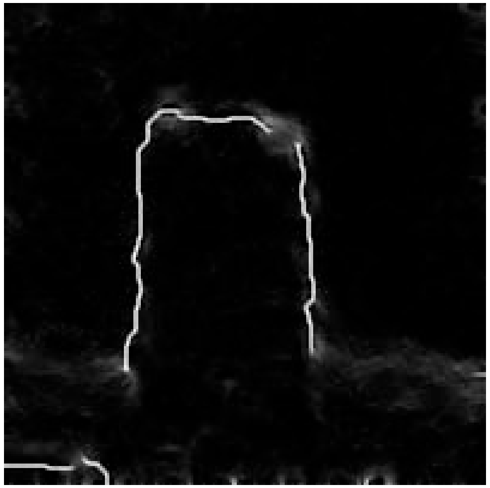
Smoothness error

Pepsi Can

201x201 (Black) Max speed: 2pix/frame

Ours

BA

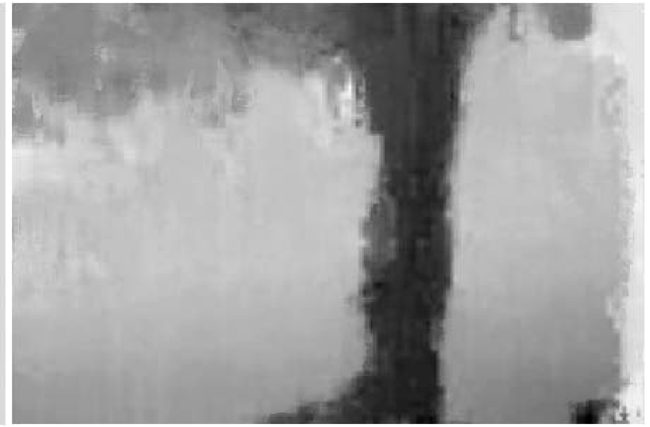


Smoothness error

FG: Flower Garden

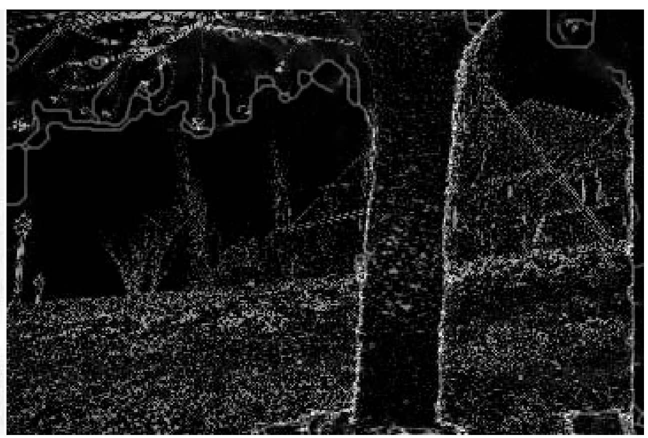
360x240 (Black)
Max speed: 7pix/frame

BA

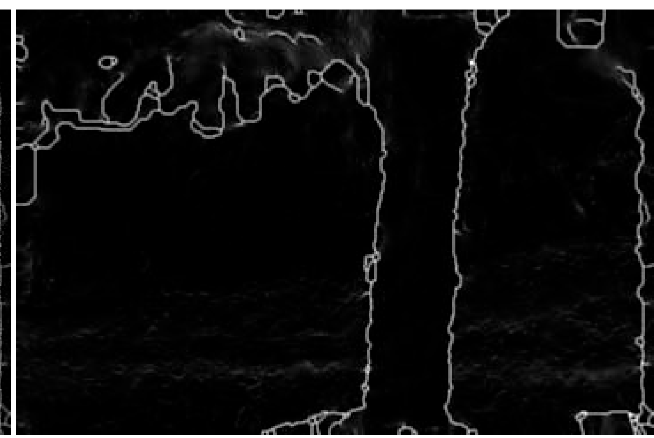


LMS

Ours



Error map



Smoothness error