## Computer Vision

# CSE 455 <br> Describing and Matching 

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## Review

- Harris Corner Dector
- SIFT Key Point Detector
- On to Descriptors


## Second Moment Matrix or Harris Matrix

$$
H=\left|\begin{array}{lll}
\mid \Sigma_{i} w_{i} I_{x} I_{x} & \Sigma_{i} w_{i} I_{x} I_{y} \mid \\
\mid \Sigma_{i} w_{i} I_{x} I_{y} & \Sigma_{i} w_{i} I_{y} I_{y} \mid
\end{array}\right|
$$

$2 \times 2$ matrix of image derivatives smoothed by Gaussian weights.


Notation:


$$
I_{y} \Leftrightarrow \frac{\partial I}{\partial y} \quad I_{x} I_{y} \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}
$$

- First compute $I_{x} I_{y}$, and $I_{x} I_{x}, I_{y} I_{y,} I_{x} I_{y}$; then apply Gaussian to each.


## From HW3: Structure matrix

- Weighted sum of gradient information
- $\left|\Sigma_{i} w_{i} l_{x}(i) l_{x}(i) \quad \Sigma_{i} w_{i} l_{x}(i) l_{y}(i)\right|$
- $\left|\Sigma_{i} w_{i} l_{x}(i)\right|_{y}(i) \quad \Sigma_{i} w_{i} l_{y}(i) l_{y}(i) \mid$


## We'll tell you how to store this!

- Use Gaussian weighting
- Eigen vectors/values of this matrix summarize the distribution of the gradients nearby
- $\lambda_{1}$ and $\lambda_{2}$ are eigenvalues
- $\lambda_{1}$ and $\lambda_{2}$ both small: no gradient
- $\lambda_{1} \gg \lambda_{2}$ : gradient in one direction
- $\lambda_{1}$ and $\lambda_{2}$ similar: multiple gradient directions, corner


## Estimating Response

$$
H=\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right] \quad \lambda_{ \pm}=\frac{1}{2}\left((a+d) \pm \sqrt{4 b c+(a-d)^{2}}\right)
$$

- A few methods we use to estimate:
- Calculate these directly from the $2 \times 2$ matrix
- $\quad \operatorname{det}(S)=a d-b c=\lambda_{1}{ }^{*} \lambda_{2}$
- $\quad$ trace(S) $=\mathrm{a}+\mathrm{d}=\lambda_{1}+\lambda_{2}$
- $\quad{ }^{* *}$ Estimate formula 1: $R=\operatorname{det}(S)-\alpha \operatorname{trace}(S)^{2}=\lambda_{1} \lambda_{2}-\alpha\left(\lambda_{1}+\lambda_{2}\right)^{2}$
- ---Estimate formula 2: $R=\operatorname{det}(S) / \operatorname{trace}(S)=\lambda_{1} \lambda_{2} /\left(\lambda_{1}+\lambda_{2}\right)$
- If these estimates are large, $\lambda_{2}$ is large


## Harris Corner Detector

- Calculate derivatives $\mathrm{I}_{\mathrm{x}}$ and $\mathrm{I}_{\mathrm{y}}$
- Calculate 3 measures $I_{x} I_{x}, I_{y} I_{y}, I_{x} I_{y}$
- Calculate weighted sums
- Want a weighted sum of nearby pixels, guess what this is?
- Gaussian!
- Estimate response
- Non-max suppression (just like Canny)




## Lowe's Pyramid Scheme



## Key point localization

s+2 difference images. top and bottom ignored. s planes searched.

- Detect maxima and minima of difference-of-Gaussian in scale space
- Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales


> For each max or min found, output is the location and the scale. above and below


## Important Point

- People just say "SIFT".
- But there are TWO parts to SIFT.

1. an interest point detector
2. a region descriptor

- They are independent. Many people use the region descriptor without looking for the points.


## Descriptors

- How are we going to describe the patch around each of the interest points we find in order to match them between images?
- We will use a very simple descriptor.
- Let's look at that first.


## Simple Normalized Descriptor

interest point

201
neighborhood around interest point

| 45 | 56 | 200 |
| :--- | :--- | :--- |
| 46 | 201 | 200 |
| 85 | 101 | 105 |

normalized neighborhood around interest point

```
156 145 1
```

156 145 1
155 0 1
155 0 1
116 100 96

```
116 100 96
```

- The simple descriptor just subtracts the center value from each of the neighbors, including itself to normalize for lighting and exposure.
- We can store this as a 1D vector to be efficient:

```
15614511550111610096
```


## Properties of our Descriptor

- Translation Invariant
- Not scale invariant
- Not rotation invariant
- Somewhat invariant to lighting changes
- Let's look at the SIFT descriptor, because it is heavily used, even without using the SIFT key point detector.
- It already solves the scale problem by computing at multiple scales and keeping track.


## Rotation invariance



- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation.


## Orientation Normalization

- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation



Once we have found the key points and a dominant orientation for each, we need to describe the (rotated and scaled) neighborhood about each.


## Full version

## SIFT descriptor

- Divide the $16 \times 16$ window into a $4 \times 4$ grid of cells ( $2 \times 2$ case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations $=128$ dimensional descriptor



Keypoint descriptor

## Full version

## SIFT descriptor

- Divide the $16 \times 16$ window into a $4 \times 4$ grid of cells
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations $=128$ dimensional descriptor



## Numeric Example

| 0.37 | 0.79 | 0.97 | 0.98 |
| :--- | :--- | :--- | :--- |
| 0.08 | 0.45 | 0.79 | 0.97 |
| 0.04 | 0.31 | 0.73 | 0.91 |
| 0.45 | 0.75 | 0.90 | 0.98 |



Orientations in each of the 16 pixels of the cell


The orientations all ended up in two bins: 11 in one bin, 5 in the other. (rough count)

## Full version

## SIFT descriptor

- Start with a $16 \times 16$ window ( 256 pixels)
- Divide the $16 \times 16$ window into a $4 \times 4$ grid of cells ( 16 cells)
- Compute an orientation histogram for each cell

- 16 cells * 8 orientations $=128$ dimensional descriptor
- Threshold normalize the descriptor:

$$
\sum_{i} d_{i}^{2}=1 \text { such that: } d_{i}<0.2
$$



## Properties of SIFT

Extraordinarily robust matching technique

- Can handle changes in viewpoint
- Up to about 30 degree out of plane rotation
- Can handle significant changes in illumination
- Sometimes even day vs. night (below)
- Fast and efficient-can run in real time
- Various code available
- http://www.cs.ubc.ca/~lowe/keypoints/



## Example



NASA Mars Rover images with SIFT feature matches
Figure by Noah Snavely

## Matching with Features

-Detect feature points in both images


## Matching with Features

-Detect feature points in both images
-Find corresponding pairs


## How do you find correspondences?

- Put the descriptors from each image into vectors and match them
- How do you compare two descriptors?
- Vector distance
- L1-Distance $(A, B)=\Sigma_{i}\left|a_{i}-b_{i}\right|$
$-L 2-\operatorname{Distance}(A, B)=\Sigma_{i} \operatorname{sqrt}\left(a_{i}{ }^{2}-b_{i}{ }^{2}\right)$
- cosine distance (angle between 2 vectors)


## Find the best matches

- For each descriptor $a$ in $A$, find its best match $b$ in B

- And store it in a vector of matches
- Note: this is abstract; see code for details.


## Matching with Features

-Detect feature points in both images
-Find corresponding pairs

- Use these matching pairs to align images.

- Larger Goal: Combine two or more overlapping images to make one larger image


Slide credit: Vaibhav Vaish

## How to do it?

- Basic Procedure

1. Take a sequence of images from the same position
(Rotate the camera about its optical center)
2. Compute transformation between second image and first
3. Shift the second image to overlap with the first
4. Blend the two together to create a mosaic
5. If there are more images, repeat
6. Take a sequence of images from the same position

- Rotate the camera about its optical center



## 2. Compute transformation between images

- Extract interest points
- Find Matches
- Compute transformation ?



## 3. Shift the images to overlap



## 4. Blend the two together to create a mosaic



## 5. Repeat for all images



## How to do it?

- Basic Procedure

1. Take a sequence of images from the same position
Rotate the camera about its optical center
2. Compute transformation between second image and first
3. Shift the second image to overlap with the first
4. Blend the two together to create a mosaic
5. If there are more images, repeat

## Compute Transformations

- Extract interest points
- Find good matches
- Compute transformation

Let's assume we are given a set of good matching interest points


## Image reprojection



- The mosaic has a natural interpretation in 3D
- The images are reprojected onto a common plane
- The mosaic is formed on this plane


## Example



## Image reprojection



- Observation
- Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another


## Motion models

- What happens when we take two images with a camera and try to align them?
- translation?
- rotation?
- scale?
- affine?

- Perspective?


## Projective transformations

- (aka homographies)

$$
\left[\begin{array}{lll}
a & b & c \\
d & e & f \\
g & h & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
1
\end{array}\right]=\left[\begin{array}{l}
u \\
v \\
w
\end{array}\right] \quad \begin{aligned}
& x^{\prime}=u / w \\
& y^{\prime}=v / w
\end{aligned}
$$



## Parametric (global) warping

- Examples of parametric warps:



## 2D coordinate transformations

- translation: $\boldsymbol{x}^{\prime}=\boldsymbol{x}+\boldsymbol{t} \quad \boldsymbol{x}=(x, y)$
- rotation: $\boldsymbol{x}^{\prime}=\boldsymbol{R} \boldsymbol{x}+\boldsymbol{t}$
- similarity: $\quad \boldsymbol{x}^{\prime}=s \boldsymbol{R} \boldsymbol{x}+\boldsymbol{t}$
- affine: $\quad \boldsymbol{x}^{\prime}=\boldsymbol{A} \boldsymbol{x}+\boldsymbol{t}$
- perspective: $\underline{\boldsymbol{x}}^{\prime} \cong \boldsymbol{H} \underline{\boldsymbol{x}} \quad \underline{\boldsymbol{x}}=(x, y, 1)$
( $\underline{\boldsymbol{x}}$ is a homogeneous coordinate)


## Image Warping

- Given a coordinate transform $\boldsymbol{x}^{\boldsymbol{\prime}}=\boldsymbol{h}(\boldsymbol{x})$ and a source image $f(x)$, how do we compute a transformed image $\boldsymbol{g}\left(\boldsymbol{x}^{\prime}\right)=\boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}))$ ?



## Inverse Warping

- Get each pixel $\boldsymbol{g}\left(\boldsymbol{x}^{\prime}\right)$ from its corresponding location $\boldsymbol{x}^{\prime}=\boldsymbol{h}(\boldsymbol{x})$ in $\boldsymbol{f}(\boldsymbol{x})$
- What if pixel comes from "between" two pixels?



## Inverse Warping

- Get each pixel $\boldsymbol{g}\left(\boldsymbol{x}^{\prime}\right)$ from its corresponding location $\boldsymbol{x}^{\prime}=\boldsymbol{h}(\boldsymbol{x})$ in $\boldsymbol{f}(\boldsymbol{x})$
- What if pixel comes from "between" two pixels?
- Answer: We already know this: resample color value from interpolated source image



## Interpolation

- Possible interpolation filters:
- nearest neighbor
- bilinear
- bicubic (interpolating)



## Motion models




2 unknowns


6 unknowns


8 unknowns

## Finding the transformation

- Translation $=2$ degrees of freedom
- Similarity $=4$ degrees of freedom
- Affine $=6$ degrees of freedom
- Homography $=8$ degrees of freedom
- How many corresponding points do we need to solve?


## Simple case: translations



How do we solve for $\left(\mathrm{x}_{t}, \mathrm{y}_{t}\right)$ ?

## Simple case: translations



$$
\begin{gathered}
\text { Displacement of match } i=\left(\mathbf{x}_{i}^{\prime}-\mathbf{x}_{i}, \mathbf{y}_{i}^{\prime}-\mathbf{y}_{i}\right) \\
\left(\mathbf{x}_{t}, \mathbf{y}_{t}\right)=\left(\frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_{i}^{\prime}-\mathbf{x}_{i}, \frac{1}{n} \sum_{i=1}^{n} \mathbf{y}_{i}^{\prime}-\mathbf{y}_{i}\right)
\end{gathered}
$$

## Simple case: translations



$$
\begin{aligned}
\mathbf{x}_{i}+\mathbf{x}_{\mathbf{t}} & =\mathbf{x}_{i}^{\prime} \\
\mathbf{y}_{i}+\mathbf{y}_{\mathbf{t}} & =\mathbf{y}_{i}^{\prime}
\end{aligned}
$$

- System of linear equations
- What are the knowns? Unknowns?
- How many unknowns? How many equations (per match)?


## Simple case: translations



$$
\begin{aligned}
\mathbf{x}_{i}+\mathbf{x}_{\mathbf{t}} & =\mathbf{x}_{i}^{\prime} \\
\mathbf{y}_{i}+\mathbf{y}_{\mathbf{t}} & =\mathbf{y}_{i}^{\prime}
\end{aligned}
$$

- Problem: more equations than unknowns
- "Overdetermined" system of equations
- We will find the least squares solution


## Least squares formulation

- For each point $\left(\mathbf{x}_{i}, \mathbf{y}_{i}\right)$

$$
\begin{aligned}
\mathbf{x}_{i}+\mathbf{x}_{\mathbf{t}} & =\mathbf{x}_{i}^{\prime} \\
\mathbf{y}_{i}+\mathbf{y}_{\mathbf{t}} & =\mathbf{y}_{i}^{\prime}
\end{aligned}
$$

- we define the residuals as

$$
\begin{aligned}
r_{\mathbf{x}_{i}}\left(\mathbf{x}_{t}\right) & =\left(\mathbf{x}_{i}+\mathbf{x}_{t}\right)-\mathbf{x}_{i}^{\prime} \\
r_{\mathbf{y}_{i}}\left(\mathbf{y}_{t}\right) & =\left(\mathbf{y}_{i}+\mathbf{y}_{t}\right)-\mathbf{y}_{i}^{\prime}
\end{aligned}
$$

## Least squares formulation

- Goal: minimize sum of squared residuals
$C\left(\mathbf{x}_{t}, \mathbf{y}_{t}\right)=\sum_{i=1}^{n}\left(r_{\mathbf{x}_{i}}\left(\mathbf{x}_{t}\right)^{2}+r_{\mathbf{y}_{i}}\left(\mathbf{y}_{t}\right)^{2}\right)$
- "Least squares" solution
- For translations, is equal to mean displacement


## Solving for translations

- Using least squares

$$
\begin{aligned}
& {\left[\begin{array}{cc}
1 & 0 \\
0 & 1 \\
1 & 0 \\
0 & 1 \\
& \vdots \\
1 & 0 \\
0 & 1
\end{array}\right]\left[\begin{array}{c}
x_{t} \\
y_{t}
\end{array}\right]=\left[\begin{array}{c}
x_{1}^{\prime}-x_{1} \\
y_{1}^{\prime}-y_{1} \\
x_{2}^{\prime}-x_{2} \\
y_{2}^{\prime}-y_{2} \\
\vdots \\
x_{n}^{\prime}-x_{n} \\
y_{n}^{\prime}-y_{n}
\end{array}\right]} \\
& \mathbf{A}_{2 n \times 2} \underset{2 \times 1}{\mathbf{t}}=\mathbf{b}_{2 n \times 1}
\end{aligned}
$$

## Least squares

$$
\mathbf{A t}=\mathbf{b}
$$

- Find $\mathbf{t}$ that minimizes

$$
\|\mathbf{A} \mathbf{t}-\mathbf{b}\|^{2}
$$

- To solve, form the normal equations

$$
\begin{gathered}
\mathbf{A}^{\mathrm{T}} \mathbf{A} \mathbf{t}=\mathbf{A}^{\mathrm{T}} \mathbf{b} \\
\mathbf{t}=\left(\mathbf{A}^{\mathrm{T}} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{b}
\end{gathered}
$$

## Affine transformations

$\left[\begin{array}{l}x^{\prime} \\ y^{\prime} \\ 1\end{array}\right]=\left[\begin{array}{lll}a & b & c \\ d & e & f \\ 0 & 0 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ 1\end{array}\right]$

- How many unknowns?
- How many equations per match?
- $x^{\prime}=a x+b y+c ; y^{\prime}=d x+e y+f$
- How many matches do we need?


## Affine transformations

- Residuals:

$$
\begin{aligned}
r_{x_{i}}(a, b, c, d, e, f) & =\left(a x_{i}+b y_{i}+c\right)-x_{i}^{\prime} \\
r_{y_{i}}(a, b, c, d, e, f) & =\left(d x_{i}+e y_{i}+f\right)-y_{i}^{\prime}
\end{aligned}
$$

- Cost function:
$C(a, b, c, d, e, f)=$

$$
\sum_{i=1}^{n}\left(r_{x_{i}}(a, b, c, d, e, f)^{2}+r_{y_{i}}(a, b, c, d, e, f)^{2}\right)
$$

## Affine transformations

- Matrix form

$$
\left[\begin{array}{cccccc}
x_{1} & y_{1} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{1} & y_{1} & 1 \\
x_{2} & y_{2} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{2} & y_{2} & 1 \\
& & \vdots & & \\
& & & \\
x_{n} & y_{n} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{n} & y_{n} & 1
\end{array}\right]\left[\begin{array}{c}
a \\
b \\
c \\
d \\
e \\
f
\end{array}\right]=\left[\begin{array}{c}
x_{1}^{\prime} \\
y_{1}^{\prime} \\
x_{2}^{\prime} \\
y_{2}^{\prime} \\
\vdots \\
\\
\\
\\
\boldsymbol{A}_{2 n \times 6}^{\prime} \\
x_{n}^{1} \\
y_{n}^{\prime}
\end{array}\right]
$$

## Solving for homographies <br> $$
\left[\begin{array}{c} x_{i}^{\prime} \\ y_{i}^{\prime} \\ 1 \end{array}\right] \cong\left[\begin{array}{lll} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{array}\right]\left[\begin{array}{c} x_{i} \\ y_{i} \\ 1 \end{array}\right]
$$

## Why is this now a variable and not just 1 ?

- A homography is a projective object, in that it has no scale. It is represented by the above matrix, up to scale.
- One way of fixing the scale is to set one of the coordinates to 1 , though that choice is arbitrary.
- But that's what most people do and your assignment code does.


## Solving for homographies

$$
\begin{array}{r}
{\left[\begin{array}{c}
x_{i}^{\prime} \\
y_{i}^{\prime} \\
1
\end{array}\right] \cong\left[\begin{array}{lll}
h_{00} & h_{01} & h_{02} \\
h_{10} & h_{11} & h_{12} \\
h_{20} & h_{21} & h_{22}
\end{array}\right]\left[\begin{array}{c}
x_{i} \\
y_{i} \\
1
\end{array}\right]} \\
x_{i}^{\prime}=\frac{h_{00} x_{i}+h_{01} y_{i}+h_{02}}{h_{20} x_{i}+h_{21} y_{i}+h_{22}} \\
y_{i}^{\prime}=\frac{h_{10} x_{i}+h_{11} y_{i}+h_{12}}{h_{20} x_{i}+h_{21} y_{i}+h_{22}}
\end{array}
$$

Why the division?
$x_{i}^{\prime}\left(h_{20} x_{i}+h_{21} y_{i}+h_{22}\right)=h_{00} x_{i}+h_{01} y_{i}+h_{02}$
$y_{i}^{\prime}\left(h_{20} x_{i}+h_{21} y_{i}+h_{22}\right)=h_{10} x_{i}+h_{11} y_{i}+h_{12}$

## Solving for homographies

$$
\begin{aligned}
x_{i}^{\prime}\left(h_{20} x_{i}+h_{21} y_{i}+h_{22}\right) & =h_{00} x_{i}+h_{01} y_{i}+h_{02} \\
y_{i}^{\prime}\left(h_{20} x_{i}+h_{21} y_{i}+h_{22}\right) & =h_{10} x_{i}+h_{11} y_{i}+h_{12}
\end{aligned}
$$

$$
\begin{aligned}
& {\left[\begin{array}{ccccccccc}
x_{i} & y_{i} & 1 & 0 & 0 & 0 & -x_{i}^{\prime} x_{i} & -x_{i}^{\prime} y_{i} & -x_{i}^{\prime} \\
0 & 0 & 0 & x_{i} & y_{i} & 1 & -y_{i}^{\prime} x_{i} & -y_{i}^{\prime} y_{i} & -y_{i}^{\prime}
\end{array}\right]\left[\begin{array}{l}
h_{00} \\
h_{01} \\
h_{02} \\
h_{10} \\
h_{11} \\
h_{12} \\
h_{20} \\
h_{21} \\
h_{22}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0
\end{array}\right]} \\
& \text { This is just for one pair of points. }
\end{aligned}
$$

## Direct Linear Transforms (n points)

Defines a least squares problem:
minimize $\|\mathrm{Ah}-0\|^{2}$

- Since $\mathbf{h}$ is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}}=$ eigenvector of $\mathbf{A}^{T} \mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points


## Direct Linear Transforms

- Why could we not solve for the homography in exactly the same way we did for the affine transform, ie.

$$
\mathbf{t}=\left(\mathbf{A}^{\mathrm{T}} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{b}
$$

## Answer from Sameer Agarwal (Dr. Rome in a Day)

- For an affine transform, we have equations of the form $A x_{i}+b$ $=y_{i}$, solvable by linear regression.
- For the homography, the equation is of the form
$H \tilde{x}_{i} \sim \tilde{y}_{i} \quad$ (homogeneous coordinates)
and the $\sim$ means it holds only up to scale. The affine solution does not hold.



## Matching features



## RAndom SAmple Consensus



## RAndom SAmple Consensus



## Least squares fit (from inliers)




## RANSAC for estimating homography

- RANSAC loop:

1. Select four feature pairs (at random)
2. Compute homography $\boldsymbol{H}$ (exact)
3. Compute inliers where $\left\|p_{i}{ }^{\prime}, \boldsymbol{H} p_{i}\right\|<\varepsilon$

- Keep largest set of inliers
- Re-compute least-squares $\boldsymbol{H}$ estimate using all of the inliers


## Simple example: fit a line

- Rather than homography H (8 numbers) fit $y=a x+b$ ( 2 numbers $a, b$ ) to 2D pairs



## Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



## Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



## Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



## Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



## Simple example: fit a line

- Use biggest set of inliers
- Do least-square fit



## What still needs to be fixed?

- The planar projections may not work so well
- Your homework has extra credit for using cylindrical projections instead.
- Here's the idea.


## Panorama algorithm:

Find corners in both images
Calculate descriptors
Match descriptors
RANSAC to find homography
Stitch together images with homography

## Stitching panoramas:

- We know homography is right choice under certain assumption:
- Assume we are taking multiple images of planar object



## In practice:



## In practice:



## In pra






## What's halppening?

What's happening? $\uparrow$

## What's happening?



## What's happening?




## What's happening?



## What's happening?




## What's happening?



## Very bad for big panoramas!



Very bad for big panoramas!


Very bad for big panoramas!


Fails :-(


## How do we fix it? Cylinders!







## How do we fix it? Cylinders!

Calculate angle and height:
$\theta=(x-x c) / f$
$h=(y-y c) / f$
Find unit cylindrical coórds:
$X^{\prime}=\sin (\theta)$
$Y^{\prime}=h$
$Z^{\prime}=\cos (\theta)$
Project to image plane:

$$
\begin{aligned}
& x^{\prime}=f X^{\prime} / Z^{\prime}+x c \\
& y^{\prime}=f Y^{\prime} / Z^{\prime}+y c
\end{aligned}
$$

$(x c, y c)=$ center of projection and $f=$ focal length of camera

## Dependant on focal length!




## $\mathrm{f}=1000$



## $\mathrm{f}=1400$



## $f=10,000$



## $f=10,000$



## Does it work?



## Does it work?



## Does it work?



## Does it work?

## 4-2

## Does it work?



## Does it work? Yay!



## Where are we?

- We are going to build a panorama from two (or more) images.
- We need to learn about
- Finding interest points
- Describing small patches about such points
- Finding matches between pairs of such points on two images, using the descriptors
- Selecting the best set of matches and saving them
- Constructing homographies (transformations) from one image to the other and picking the best one
- Stitching the images together to make the panorama

