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Homework 1

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Assigned: Friday, October 12, 2001
Due: Friday, October 26, 2001

DIRECTIONS

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to talk over the problems with classmates, but please *answer the questions on your own*.

NAME: ________________________________

Problem 1. Short answer (14 points, 2 each)
Provide a short answer (typically one or two sentences) to each of the following questions. In each case, you must clearly justify your answer.

a. How are the electrons coming out of the red, green, and blue electron guns of a color monitor different?

b. At 24 bits per pixel (True Color), can a monitor produce every color perceptible to the human eye?

c. Is the median filter a convolution filter?

d. If you convolve an image with the Laplacian filter, will you typically get an image that looks about the same except a bit sharper?

e. Given red, green, and blue lights, can every color in the visible spectrum be matched (with respect to human vision) by adjusting the intensities of the lights? Explain.

f. How does the use of the second derivative in edge detection improve the results?

g. Why do we use a 4x4 matrix for transformations in 3-space?

Problem 2. Image Filters (16 points)

Each of the matrices below is actually a convolution filter for image processing. In addition, we have included a median filter. Next to each filter, identify the image (next page) that would result from applying the filter. In addition, write all of the characteristics from the following list that apply:

- mean blurring
- gaussian blurring
- edge-preserving blurring
- blurring in x
- blurring in y
- gradient in *x*
- gradient in *y*
- edge enhancing
- rotating
- translating
- identity (no effect).

(a) $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

Image: ____

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(b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Image: ____

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(c) $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Image: ____

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(d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Image: ____

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(e) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Image: ____

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(f) $\frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

Image: ____

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(g) $\frac{1}{5}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Image: ____

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(h) Median filter over 4x4 region.

Image: ____

____

____

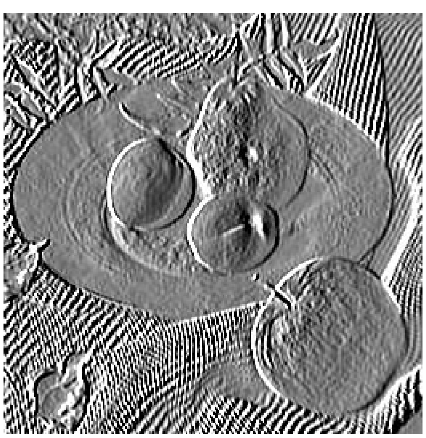
1. Original image

2

3

4

5



6 [after adding 128 to each pixel]

7

8 [after adding 128 to each pixel]

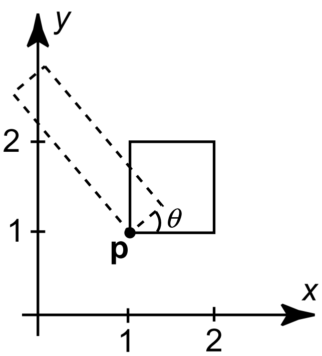
Problem 3. Color (10 points)
Two identical objects are in a single room under identical lighting. Someone viewing the two objects makes the observation that they both appear to have the same color. In all cases the angle of viewing of the objects can be treated as the same.

a) Is it possible that the objects give off different spectral emissions? Explain.

b) The lighting in the room containing the objects is then changed. Both objects are still lit identically relative to each other, but that lighting has changed relative to the original scene. Is it possible that the observer would now perceive the objects to differ in color from each other? Why?

c) The original lighting is restored and the viewer is given a pair of colored glasses to wear. Again both objects are viewed simultaneously through the glasses, so that the effects of the colored glasses are visited equally on both objects. Under these conditions, is it possible that the observer will find that the two objects differ in color from each other? Why?

Problem 4. Transformations (15 points)
Let's assume you want to perform a scale, **S**(1/2, 2), and then perform a rotation, **R**(θ), with respect to a point **p** = (1,1). The figure below shows how a unit cube (solid) sitting at **p** would be transformed into a rotated rectangle (dotted) by this process.



Write out the sequence of matrices, in terms of S(a,b), R(θ), and translation, T(c,d), that would be needed to perform this transformation. You may leave all matrices in symbolic form – do **not** expand them into 3x3 matrices. θ is the only variable, you must fill in all other arguments as numbers.

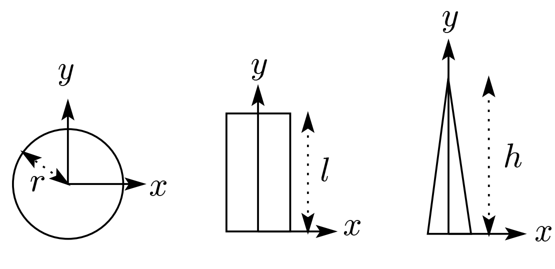
Problem 5. Affine Transformations (15 points)

Using the Euler angle rotation matrices Rx(α),Ry(α) and Rz(α) (parameterized by the angle α) as building blocks, specify how one could build a transformation matrix which rotates around the arbitrary axis vector **v**=(x,y,z) by the angle θ. In words and drawings, describe all parts of the construction. You don't need to compute exact formulas for the rotation angles, but you must describe how to compute each of the rotation angles.

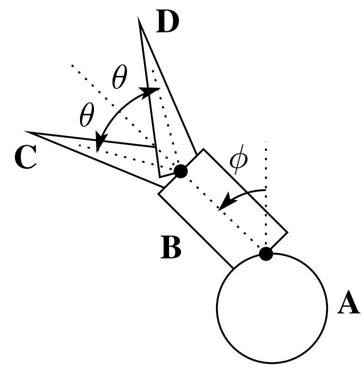
Problem 6. Hierarchies (15 points)
Suppose you want to model the pinching figure shown below. The pincher is made of four parts, labeled **A**, **B**, **C**, and **D** and each part is drawn as one of three primitives as given below.

The following transformations are also available to you:

- R(t) – rotate by t degrees (counter clockwise)
- T(a,b) – translate by (a,b)



Primitives



Pincher diagram

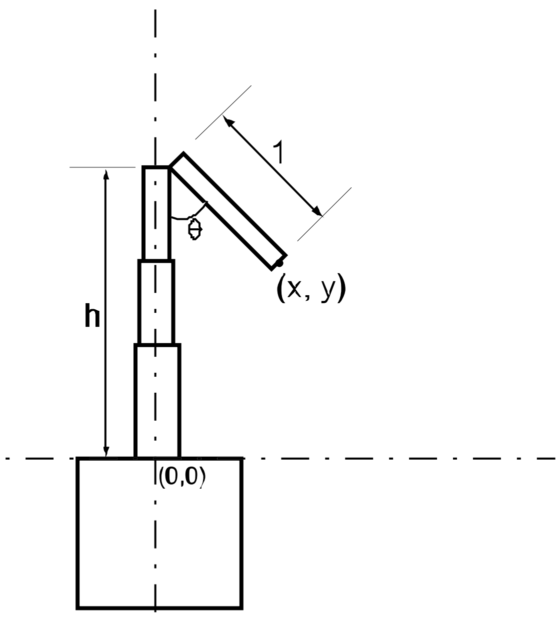
a) Construct a tree to specify the pincher that is rooted at A. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that order is important!

b) Write out the full transformation expression for the part labeled **C**.

Problem 7. Inverse Kinematics (15 points)

In the hierarchies we have studied in class so far, we have dealt only with forward kinematics; that is, if we want to determine a particular position of a limb, we specify joint angles for all the joints such that the limb is positioned in the way we want.

In certain cases, however, it would be useful to instead specify coordinate positions of the limbs; for example, if we wanted a character to pick up a coffee cup, it would be useful to specify the hand position in the same (x, y, z) world coordinates that the cup's position is defined in. This is known as inverse kinematics, and in order for it to work properly, we must be able to determine the joint angles that would result in the correct hand position. In this problem, we will explore such a solver for a simple case.



The robot arm at left has a hierarchy rooted at (0, 0). The upper arm (labeled h) is telescoping, while the lower arm has the fixed length 1. Assume also that there are no constraints on the length of the telescoping arm. Answer the following questions:

a) Suppose we set the point (x, y) to be equal to $\left(\frac{1}{2}, 2-\frac{\sqrt{3}}{2}\right)$. What would the angle θ and the length h be then?

b) Is this the only solution? Why?

c) Suppose (x, y) is some arbitrary point, and invert the equations. That is, solve for the joint angle θ and the upper arm length h in terms of the (x, y) coordinates.