Color

CSE 457, Autumn 2003

Graphics

http://www.cs.washington.edu/education/courses/457/03au/

Readings and References

Readings

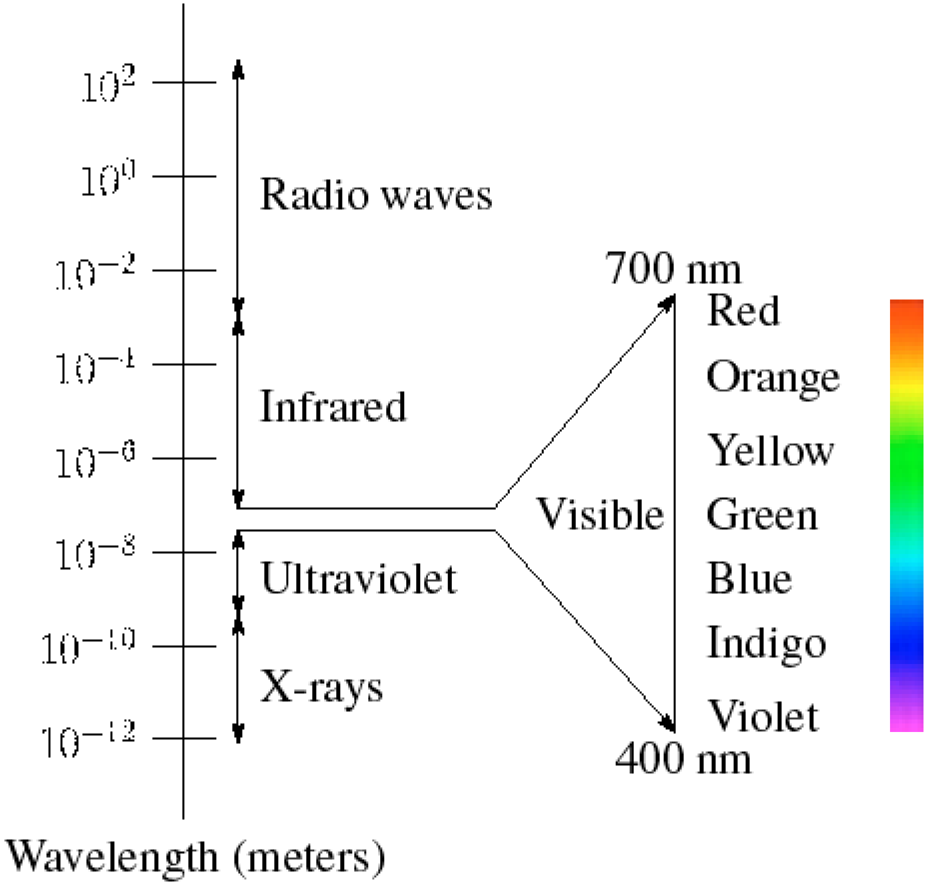
- Chapter 15, *3D Computer Graphics*, Watt

Other References

- Chapter 4, pp. 69-97, *Foundations of Vision*, Wandell

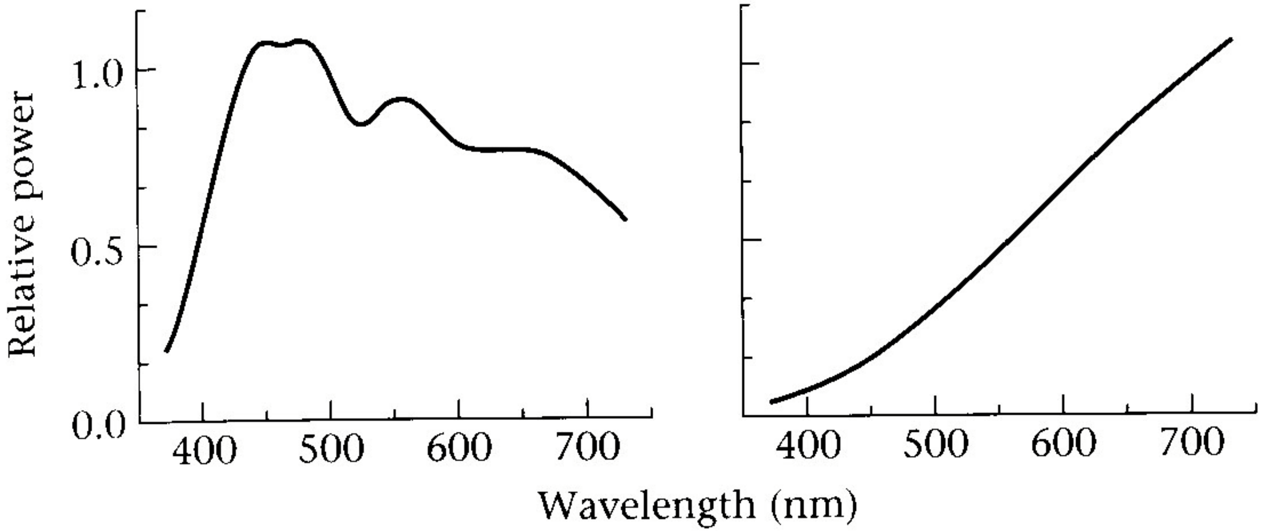
The radiant energy spectrum

Wave theory allows a nice arrangement of electromagnetic radiation (EMR) according to wavelength

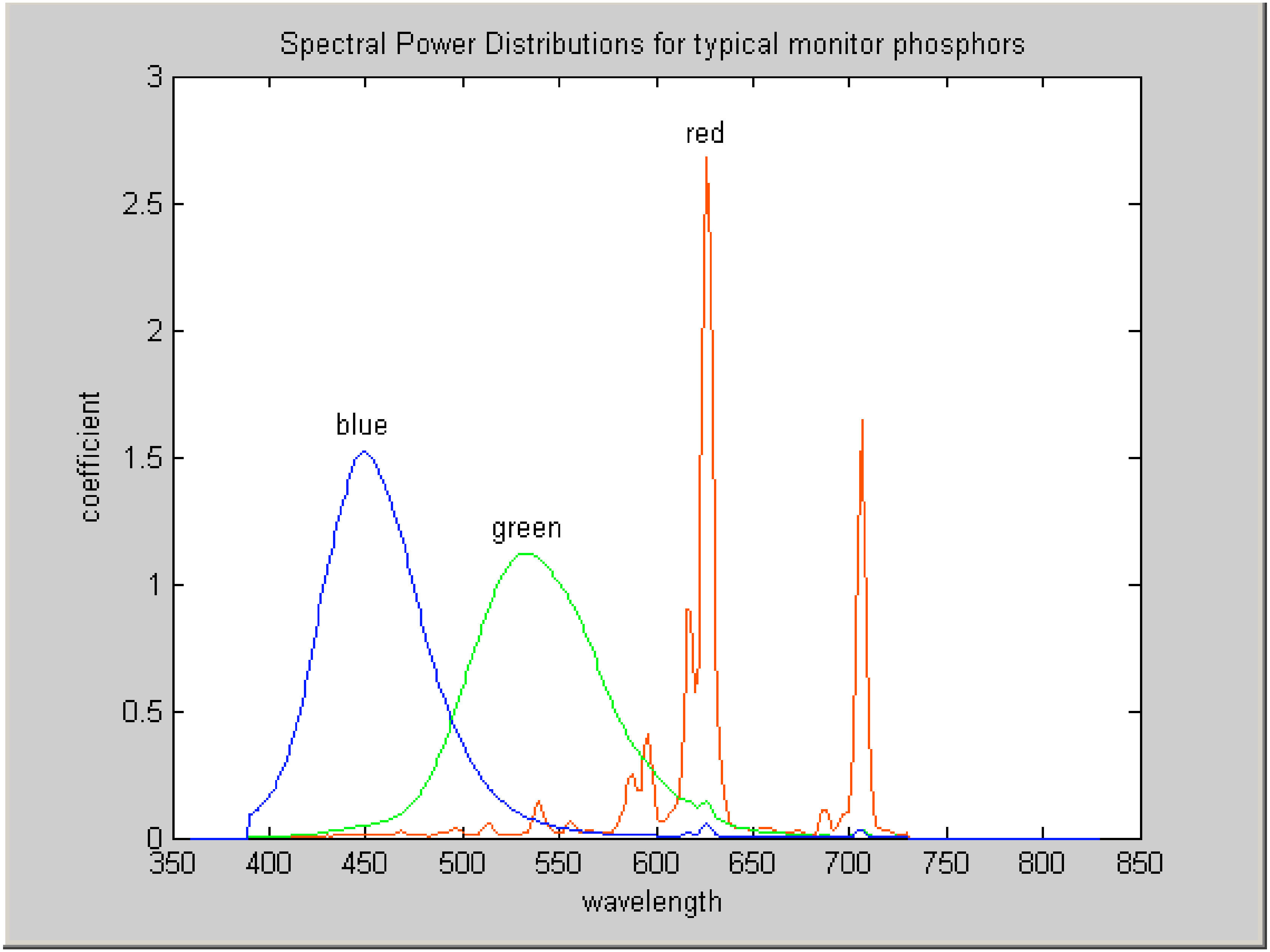


Emission spectra

- The basic nature of a light source can be described by its Spectral Power Distribution
  - » The SPD gives the energy at each wavelength



Emission spectra for daylight and a tungsten lightbulb (Wandell, 4.4)



Data from Wandell, Applied Vision and Imaging Systems , http://white.stanford.edu/~brian/

What is color?

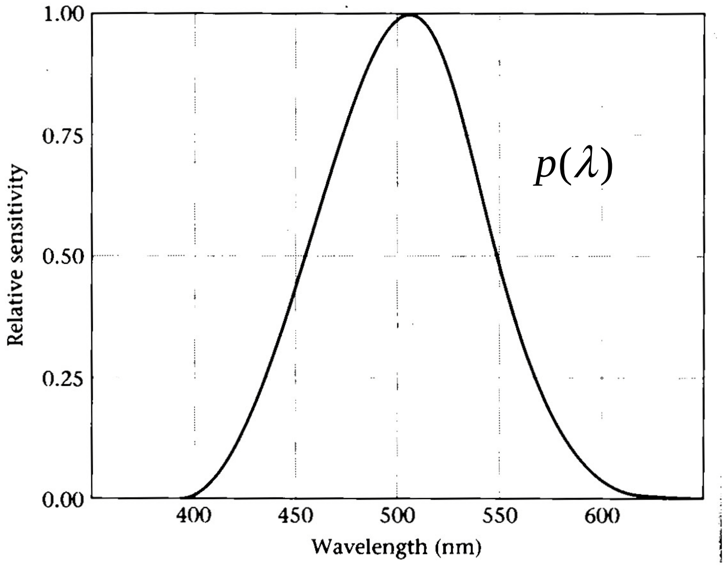
- The eyes and brain turn an incoming emission spectrum into a (small) discrete set of values.
- The signal sent to our brain is *interpreted* as *color*.
  - » *When I use a word it means just what I choose it to mean, neither more nor less.* - H. Dumpty, 1871
  - » Color is not the same as Spectral Power Distribution
- Color science asks some basic questions:
  - » When are two colors alike?
  - » How many pigments or primaries does it take to match another color?

Rod Photopigments

- **Photopigments** are the chemicals in the rods and cones that react to light. Can respond to a single photon!

Rods contain **rhodopsin**, which has peak sensitivity at about 500nm.

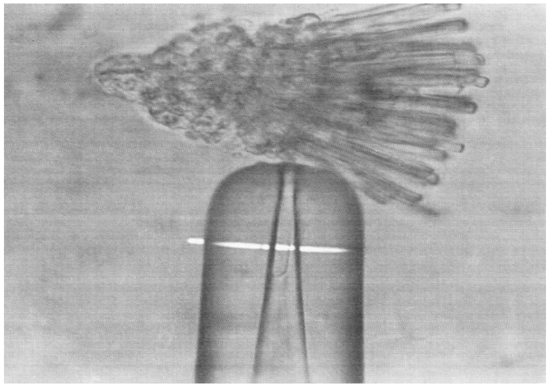
Rods are active under low light levels, i.e., they are responsible for **scotopic** vision.

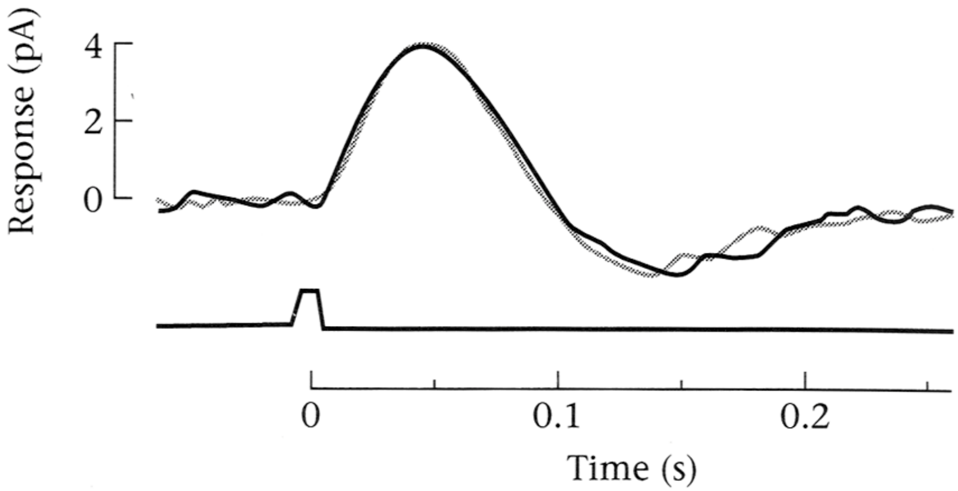


Spectral sensitivity function for rods (Wandell ,4.6)

Univariance

Principle of univariance: For any single photoreceptor, *no* information is transmitted describing the wavelength of the photon.





Photocurrents measured for two light stimuli: 550nm (solid) and 659 nm (gray). The brightnesses of the stimuli are different, but the shape of the response is the same. (Wandell 4.17)

The color matching experiment

We can construct an experiment to see how to match a given test light using a set of lights called **primaries** with power control knobs.

The test light SPD function is

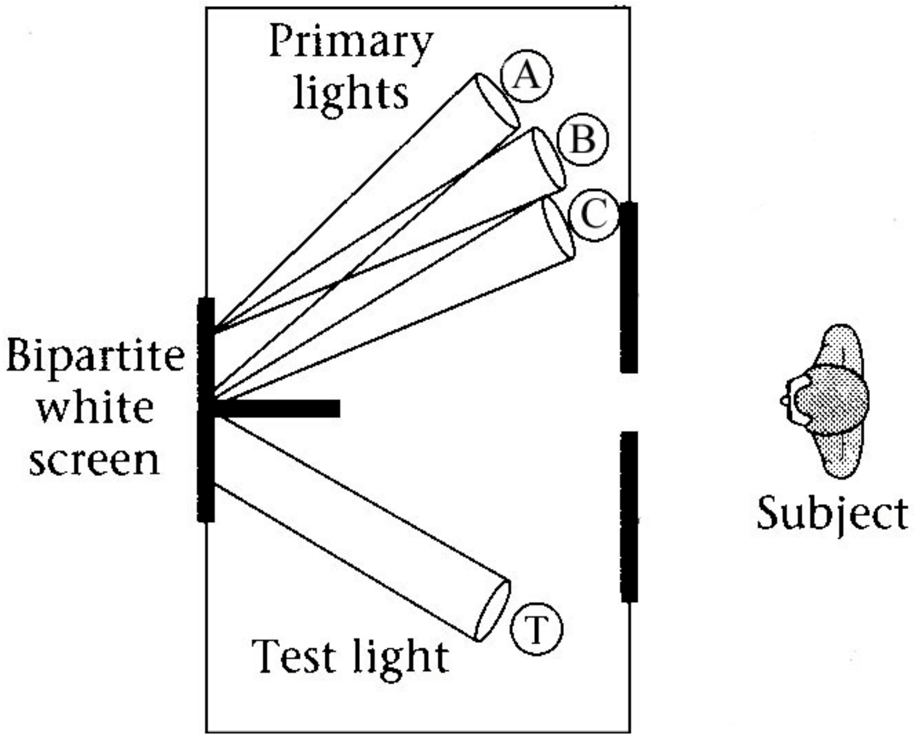
$t(\lambda)$

The primary light SPDs are

$a(\lambda)$, $b(\lambda)$, $c(\lambda)$, …

The power knob settings are

A, B, C, …



The color matching experiment (Wandell, 4.10)

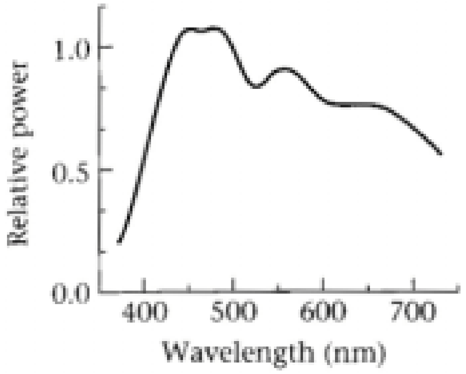
Rods and "color" matching

A rod responds to a spectrum through its spectral sensitivity function, *p(λ)*. The rod response to a test light with SPD *t(λ)* is

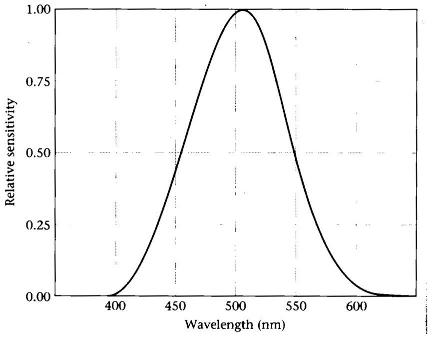
$$P_t = \int t(\lambda)p(\lambda)d\lambda$$

Notice that P_t is a scalar value ∈ [0,max].

$$t(\lambda)$$



$$p(\lambda)$$



Rod response to multiple primaries

The rod response to primary lights with SPD *a*(λ), *b*(λ), *c*(λ) is

$$P_{abc} = \int \left(A \cdot a(\lambda) + B \cdot b(\lambda) + C \cdot c(\lambda) \right) p(\lambda) d\lambda$$

Notice that P_{abc} is a scalar value $\in$ [0,max].

So then what does it mean to “match a test light” using scotopic vision only?

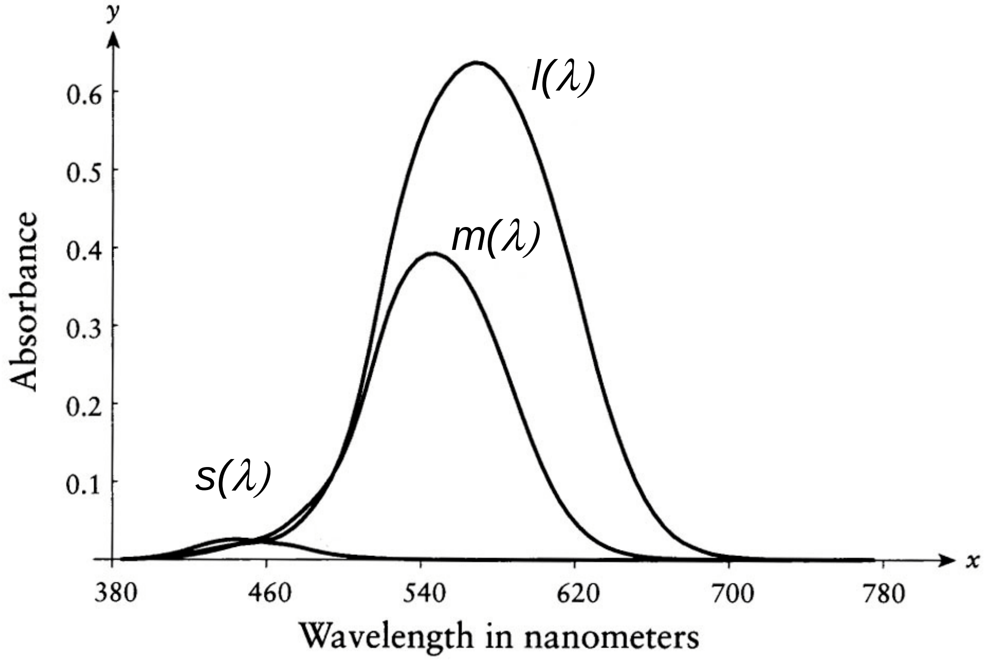
How many primaries are needed to match the test light?

What does this tell us about rod color discrimination?

Cone photopigments

Cones are active under higher light levels, i.e., they are responsible for **photopic** vision.

Cones come in three varieties: L, M, and S.



Cone photopigment absorption (Glassner, 1.1)

Cones and color matching

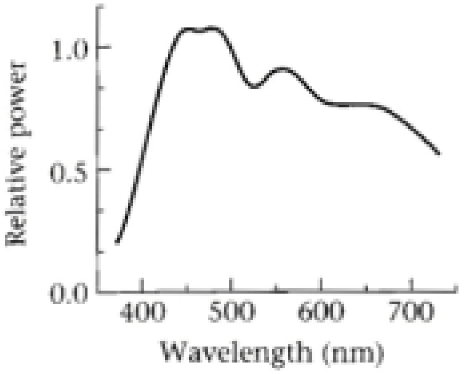
A cone responds to a spectrum through its spectral sensitivity function, $p(\lambda)$. The cone responses to a test light with SPD $t(\lambda)$ are

$$L = \int t(\lambda) l(\lambda) d\lambda$$

$$M = \int t(\lambda) m(\lambda) d\lambda$$

$$S = \int t(\lambda) s(\lambda) d\lambda$$

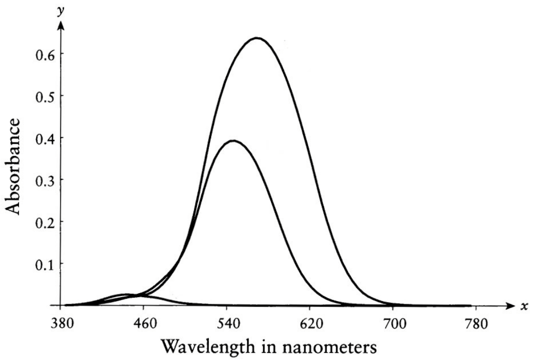
$t(\lambda)$



$l(\lambda)$

$m(\lambda)$

$s(\lambda)$



This is a projection from ∞-space to 3-space

Cone response to multiple primaries

Consider again three primaries, $a(\lambda)$, $b(\lambda)$, $c(\lambda)$, with three emissive power knobs, A, B, C.

The three knobs and three primaries create spectra of the form:

$$SPD_{abc} = A \cdot a(\lambda) + B \cdot b(\lambda) + C \cdot c(\lambda)$$

The m-cone response M to this combination of primary lights is

$$\begin{aligned} M_{abc} &= \int \left(Aa(\lambda) + Bb(\lambda) + Cc(\lambda) \right) m(\lambda)\, d\lambda \\ &= \int Aa(\lambda) m(\lambda)\, d\lambda + \int Bb(\lambda) m(\lambda)\, d\lambda + \int Cc(\lambda) m(\lambda)\, d\lambda \\ &= A \int a(\lambda) m(\lambda)\, d\lambda + B \int b(\lambda) m(\lambda)\, d\lambda + C \int c(\lambda) m(\lambda)\, d\lambda \\ &= AM_a + BM_b + CM_c \end{aligned}$$

Color matching, cont'd

We end up with similar relations for all the cones:

$$L_{abc} = AL_a + BL_b + CL_c$$
$$M_{abc} = AM_a + BM_b + CM_c$$
$$S_{abc} = AS_a + BS_b + CS_c$$

We can re-write this as a matrix and solve for the knob settings:

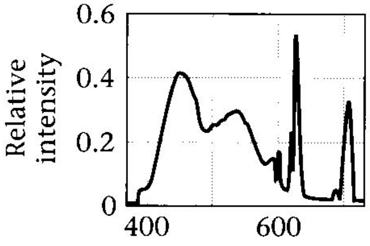
$$\begin{bmatrix} L_{abc} \\ M_{abc} \\ S_{abc} \end{bmatrix} = \begin{bmatrix} L_a & L_b & L_c \\ M_a & M_b & M_c \\ S_a & S_b & S_c \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} L_a & L_b & L_c \\ M_a & M_b & M_c \\ S_a & S_b & S_c \end{bmatrix}^{-1} \begin{bmatrix} L_{abc} \\ M_{abc} \\ S_{abc} \end{bmatrix}$$

We can find (A,B,C) to match any (L,M,S) and hence any color!

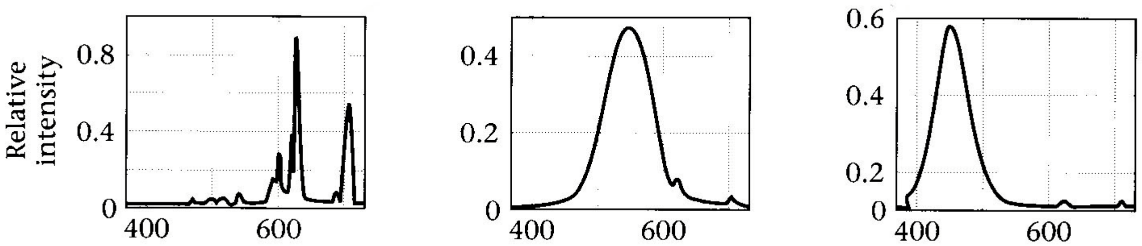
Negative values can be accommodated by adding the primary to the test light.

Choosing Primaries

The primaries could be three color (monochromatic) lasers, but there is no reason why they have to be. They are just a way to form basis vectors in LMS space. They can be non-monochromatic, e.g., monitor phosphors:



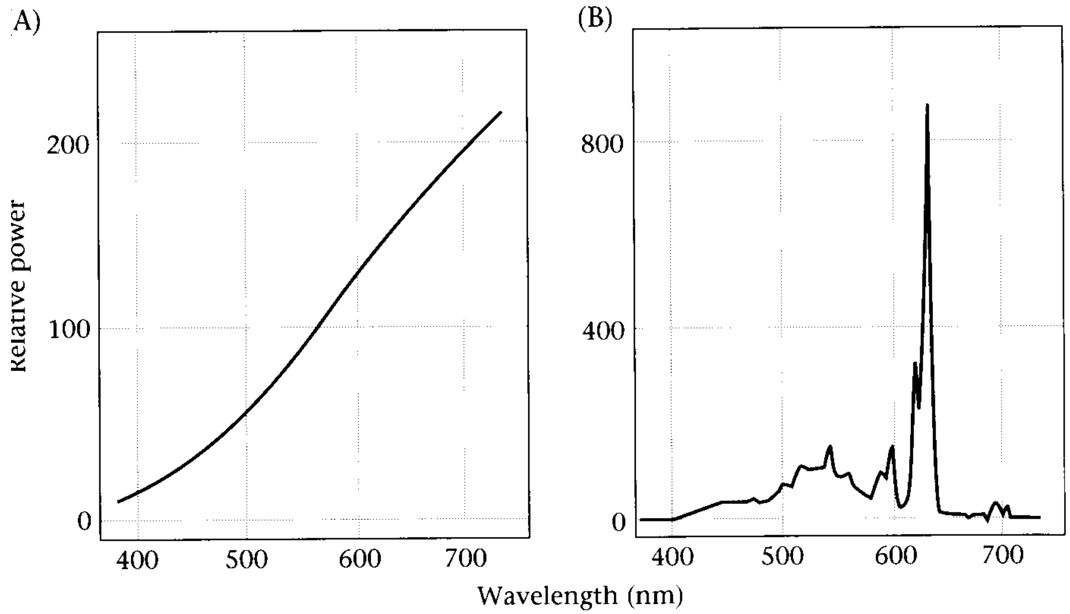
$$SPD_{rgb} = Rr(\lambda) + Gg(\lambda) + Bb(\lambda)$$



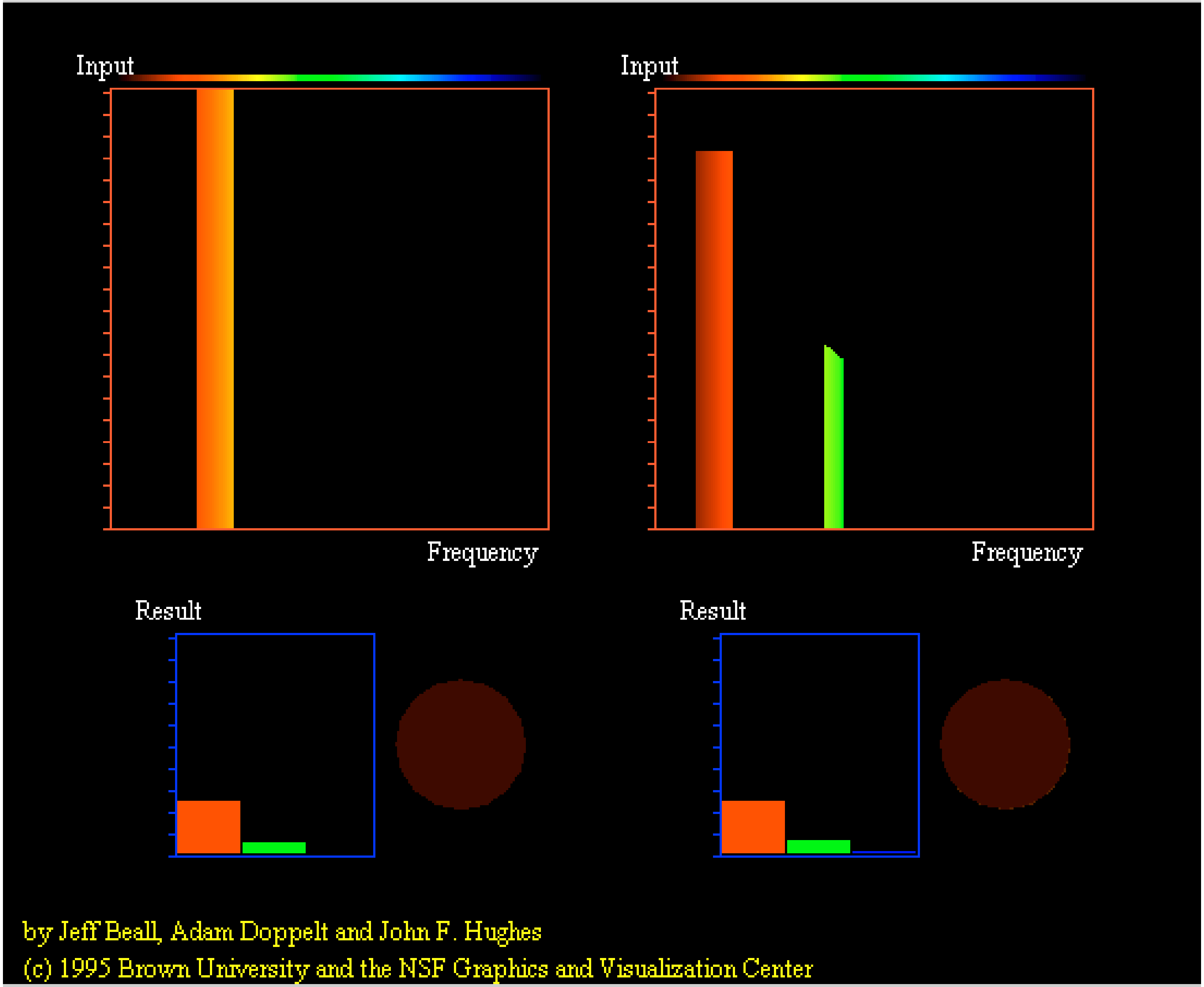
Emission spectra for RGB monitor phosphors (Wandell B.3)

Emission Spectrum *is not* Color

- The light spectrum is infinite dimensional!
- Different light sources can evoke exactly the same perceived colors. Such lights are called **metamers**.



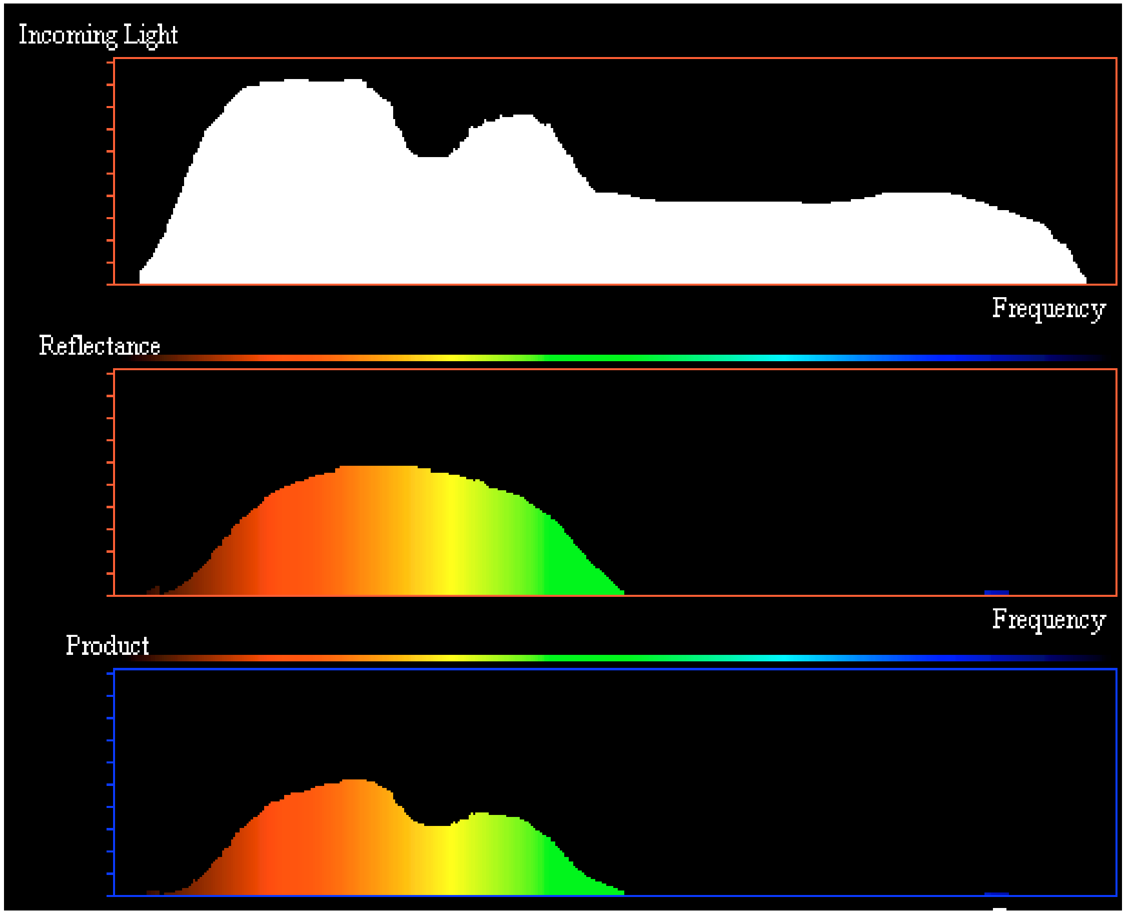
A dim tungsten bulb and an RGB monitor set up to emit a metameric spectrum (Wandell 4.11)



by Jeff Beall, Adam Doppelt and John F. Hughes

Colored Surfaces

So far, we've discussed the colors of lights. How do *surfaces* acquire color?



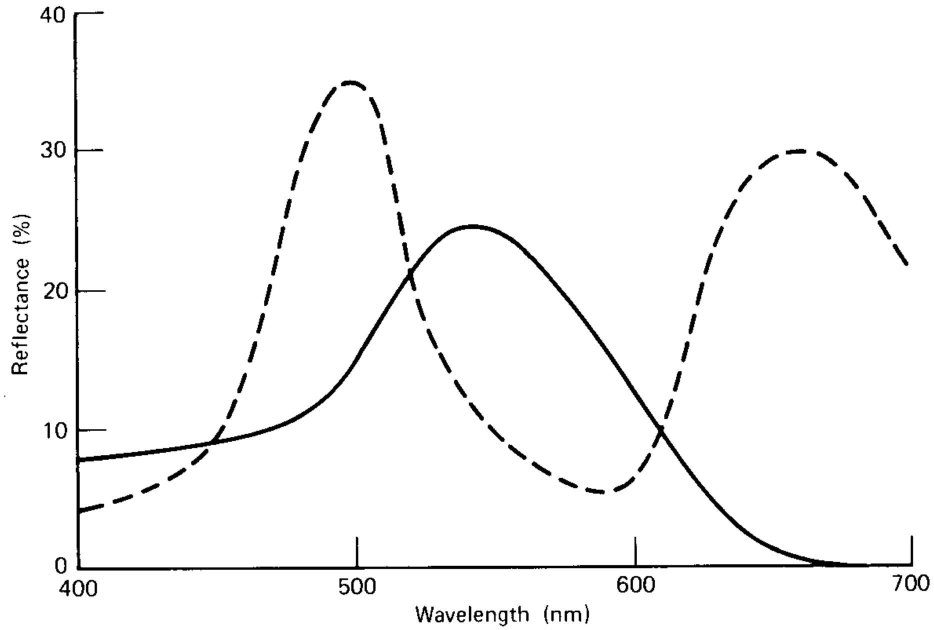
SPD of the incoming light

A surface's **reflectance**, $\rho(\lambda)$, is its tendency to reflect incoming light across the spectrum.

The incoming light is multiplied by the reflectance (thereby "subtracting" or removing color).

$$I(\lambda) = \rho(\lambda)t(\lambda)$$

Subtractive Metamers



Surfaces that are metamers under only some lighting conditions (Wasserman 3.9)

Reflectance adds a whole new dimension of complexity to color perception.

The solid curve appears green indoors and out. The dashed curve looks green outdoors, but brown under incandescent light.

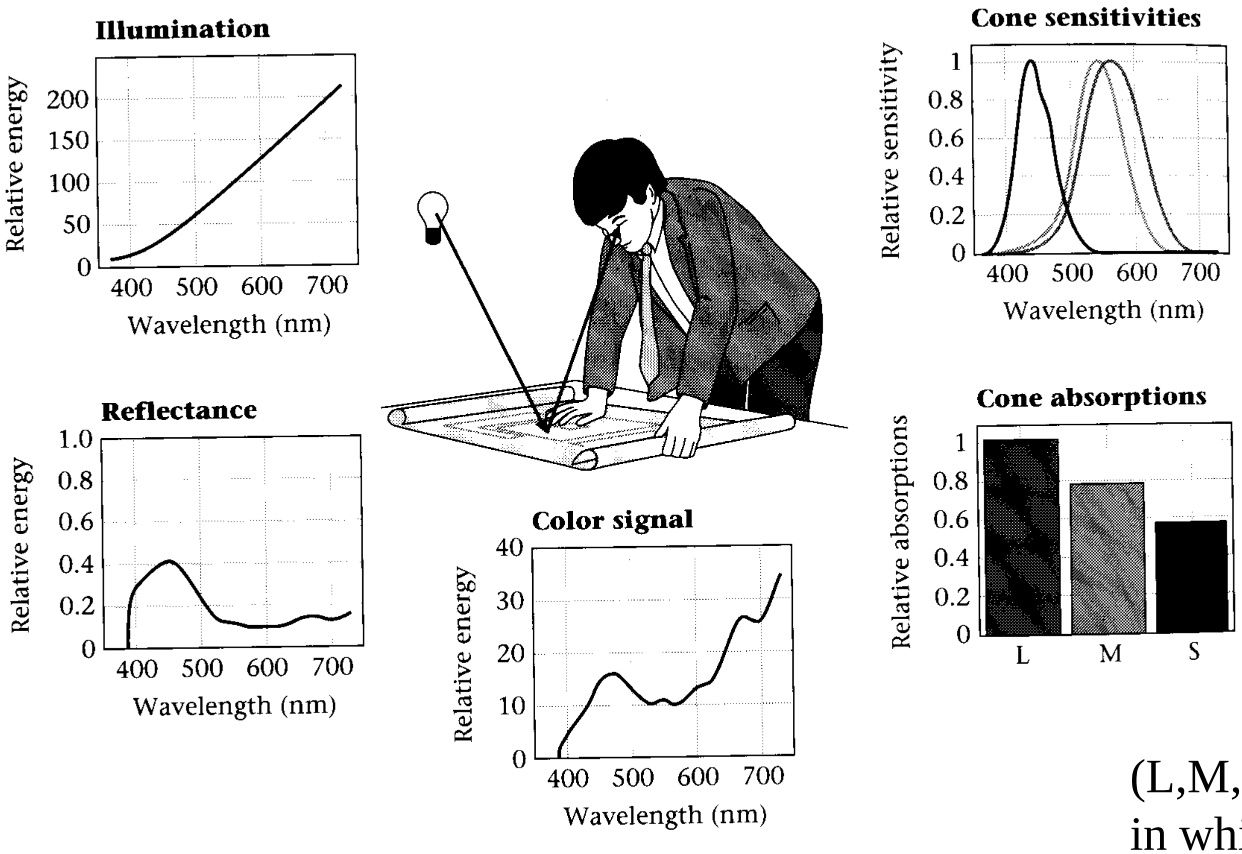
Lighting design

When deciding the kind of “feel” for an architectural space, the spectra of the light sources is critical.

Lighting design centers have displays with similar scenes under various lighting conditions.

Lighting Design Lab in Seattle: http://lightingdesignlab.com/

Illustration of Color Appearance

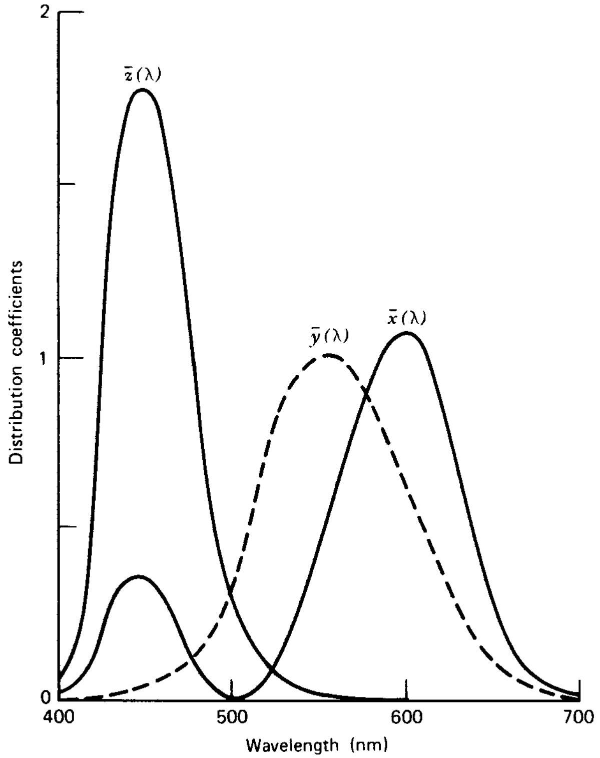


(L,M,S) is the 3-D space
in which we perceive colors

How light and reflectance become cone responses (Wandell, 9.2)

The CIE *XYZ* System

A standard created in 1931 by CIE, defined in terms of three color matching functions.



The XYZ color matching functions (Wasserman 3.8)

These functions are related to the cone responses as roughly:

$$\bar{x}(\lambda) \approx k_1 s(\lambda) + k_2 l(\lambda)$$
$$\bar{y}(\lambda) \approx k_3 m(\lambda)$$
$$\bar{z}(\lambda) \approx k_4 s(\lambda)$$

CIE Coordinates

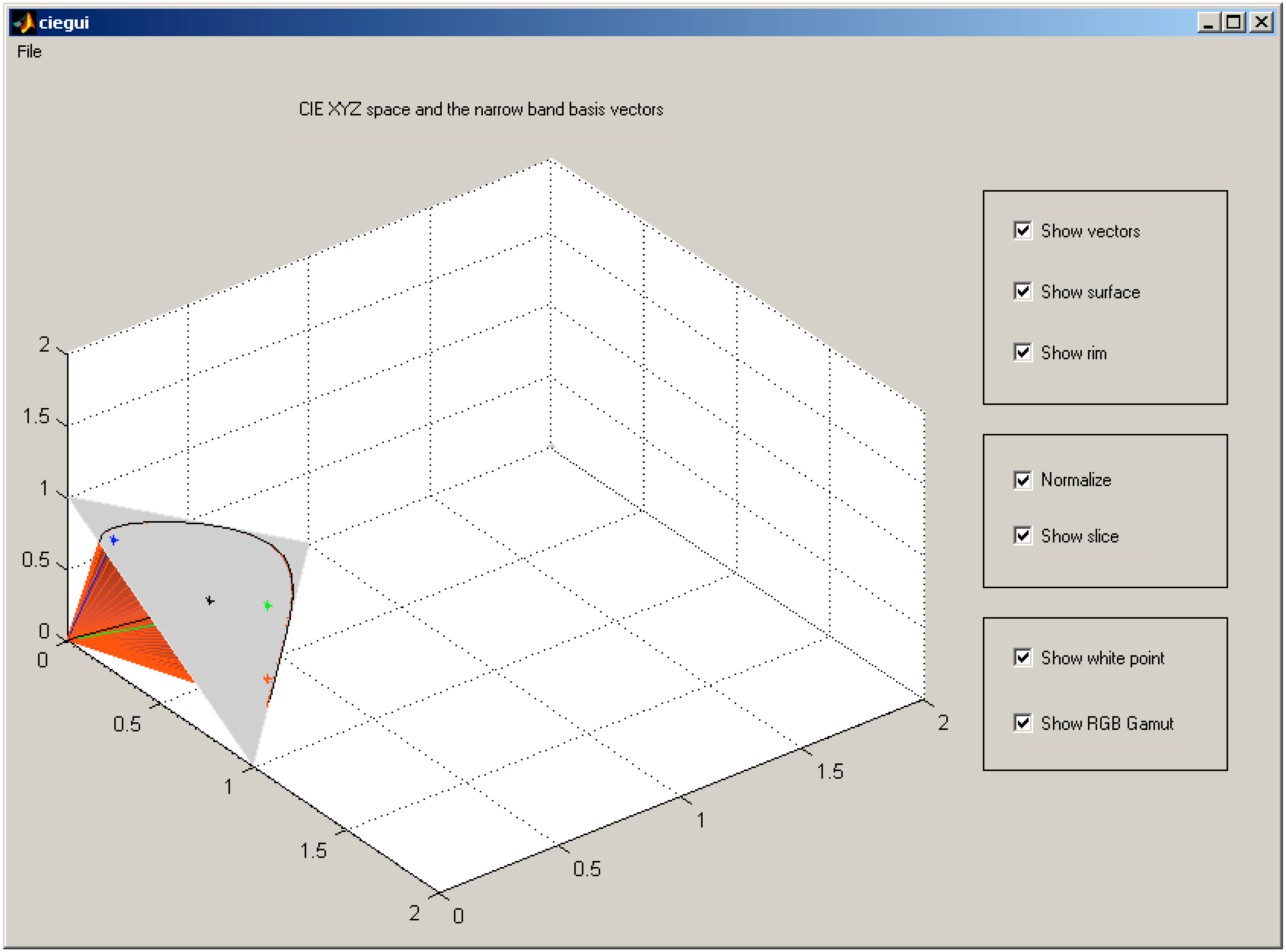
Given an emission spectrum, we can use the CIE matching functions to obtain the *X*, *Y* and *Z* coordinates.

$$X = \int \bar{x}(\lambda)t(\lambda)d\lambda \qquad Y = \int \bar{y}(\lambda)t(\lambda)d\lambda \qquad Z = \int \bar{z}(\lambda)t(\lambda)d\lambda$$

Given the equations on the preceding page, the XYZ coordinates are closely related to LMS responses.

The XYZ space is a linear transformation of LMS space and we can think of it exactly the same way. The CIE primaries are the basis vectors for a 3-D space, and (X,Y,Z) are the coordinates of a particular color in this space.

Using the color matching functions, we can map any SPD to a particular point in this space and that point represents the perceived color for that SPD.

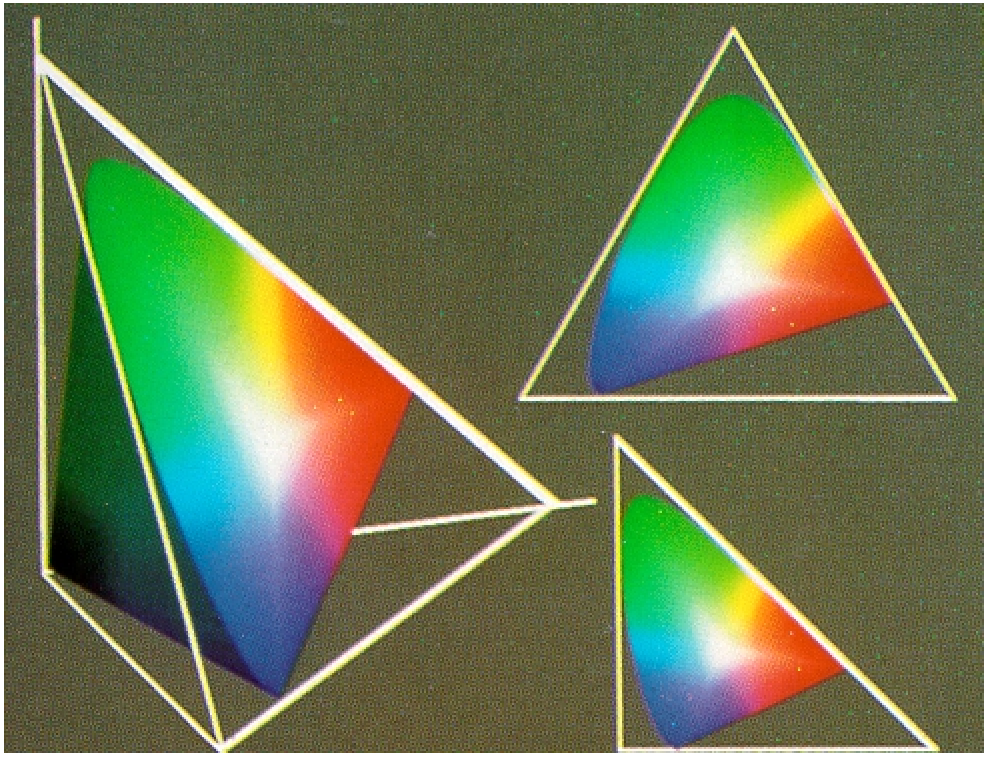


☑ Show vectors
☑ Show surface
☑ Show rim

☑ Normalize
☑ Show slice

☑ Show white point
☑ Show RGB Gamut

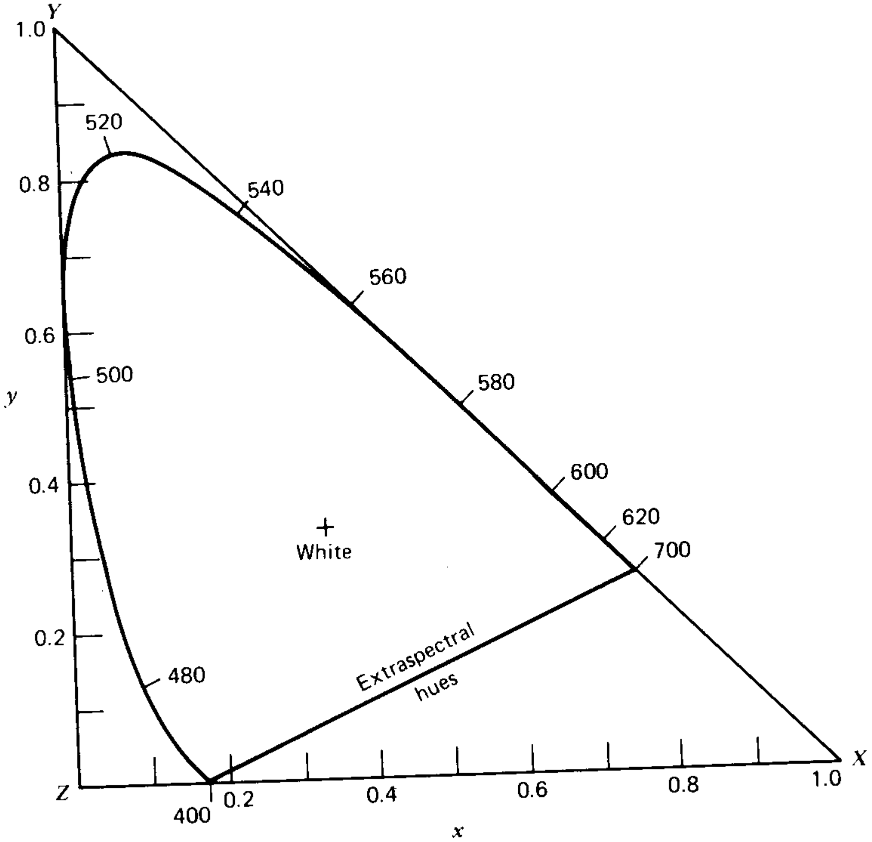
The CIE Colour Blob



Different views of the CIE color space (Foley II.1)

The CIE Chromaticity Diagram

- A projection of the plane $X+Y+Z=1$.
- Each point is a chromaticity value, which depends on **dominant wavelength**, or **hue**, and **excitation purity**, or **saturation**.



The chromaticity diagram (a slice through CIE space, Wasserman 3.7)

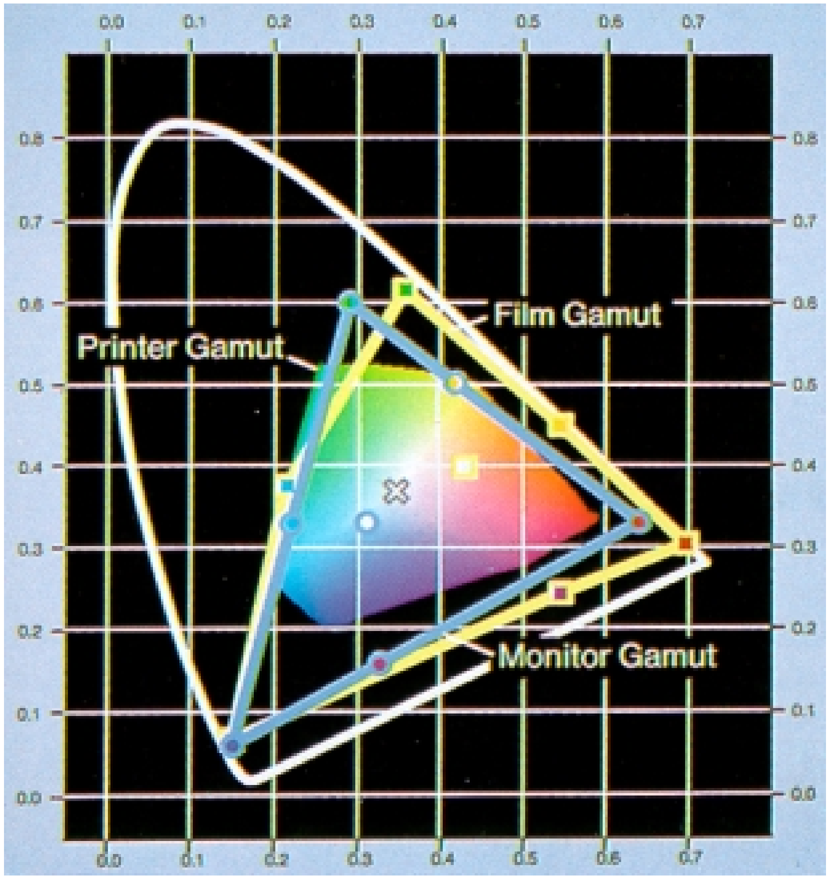
More About Chromaticity

- Narrow-band SPDs (single frequency lights) map to points around the perimeter of the chromaticity blob
  - » A color’s dominant wavelength is where a line from white through that color intersects the perimeter.
  - » Some colors, called *nonspectral* color’s, don’t have a dominant wavelength because their perimeter color cannot be obtained with a single narrow-band input.
- Excitation purity is measured in terms of a color’s position on the line to its dominant wavelength.
- Complementary colors lie on opposite sides of white, and can be mixed to get white.

Gamuts

Not every output device can reproduce every color. A device's range of reproducible colors is called its **gamut**.

The technology of the device determines the basis vectors of the subspace it can address.



Gamuts of a few common output devices in CIE space (Foley, II.2)

Color Spaces for Computer Graphics

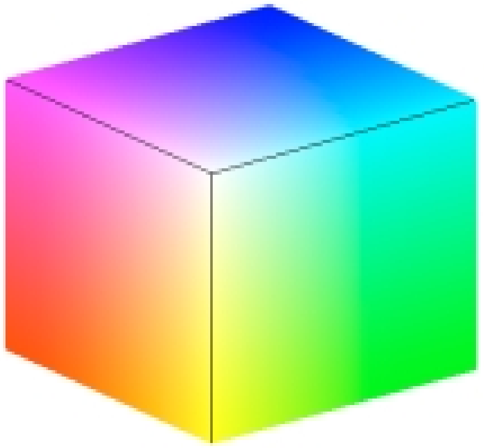
In practice, there’s a set of more commonly-used color spaces in computer graphics:

- RGB for display
- CMY (or CMYK) for hardcopy
- HSV for user selection
- YIQ for broadcast

RGB

Perhaps the most familiar color space, and the most convenient for display on a CRT.

What does the RGB color space look like?



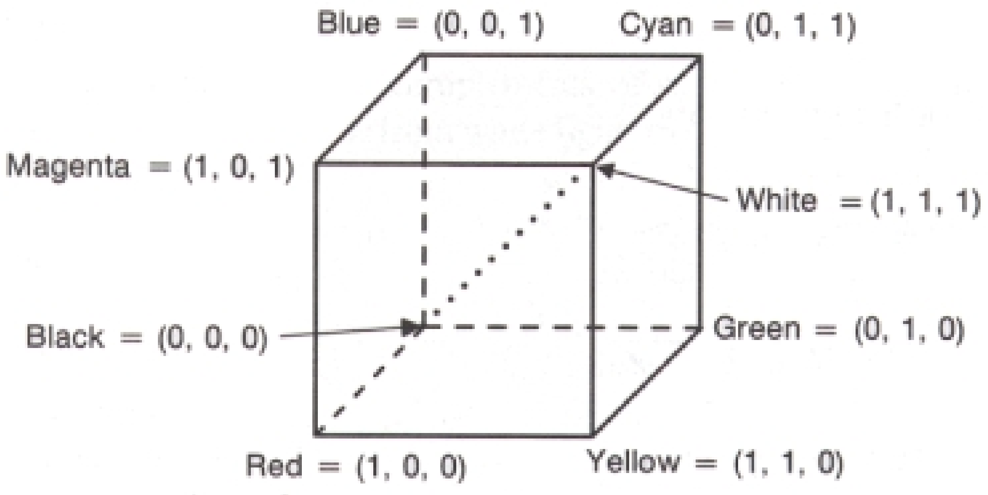


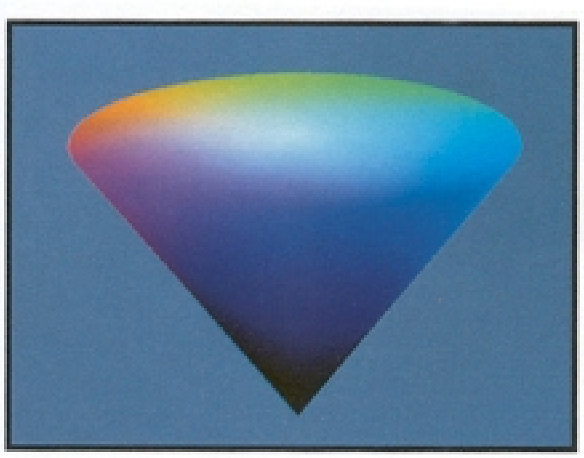
Fig. 13.28 The RGB cube. Grays are on the dotted main diagonal.

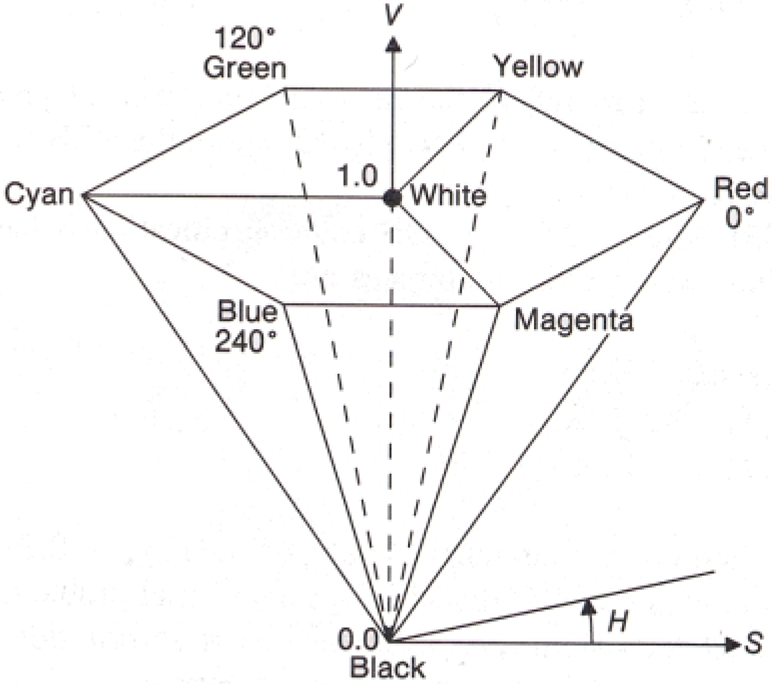
Foley

HSV

More natural for user interaction, corresponds to the artistic concepts of tint, shade and tone.

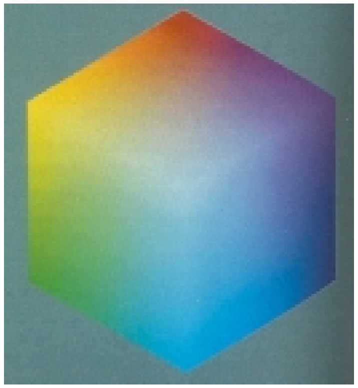
The HSV space looks like a cone:





Foley Fig 13.30

CMY



A subtractive color space used for printing.

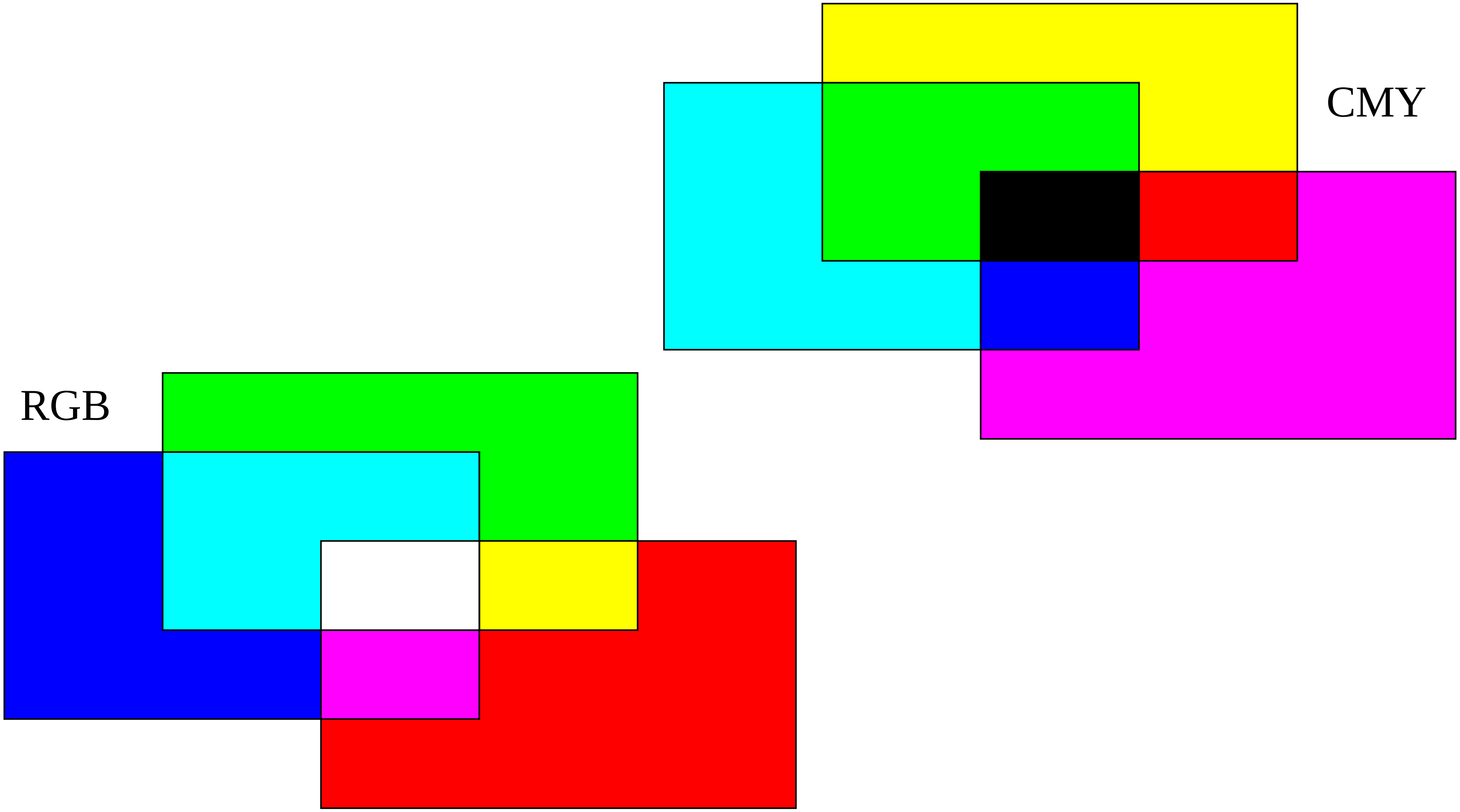
Involves three subtractive primaries:

- Cyan - subtracts red
- Magenta - subtracts green
- Yellow - subtracts blue

Mixing two pigments subtracts their opposites from white.

CMYK adds blacK ink rather than using equal amounts of all three.

RGB vs. CMY



YIQ

Used in TV broadcasting, YIQ exploits useful properties of the visual system.

- Y - luminance (taken from CIE)
- I - major axis of remaining color space
- Q - remaining axis

YIQ is broadcast with relative bandwidth ratios 8:3:1

Why do we devote a channel to luminance?

Summary

Here’s what you should take home from this lecture:

- All the **boldfaced terms**.
- How to compute cone responses
- The difference between emissive and reflective color
- What the CIE XYZ color standard and chromaticity diagram are
- The color spaces used in computer graphics