Computer Graphics	**Prof. Brian Curless**
CSE 457	**Spring 2004**

Homework #1

Displays, Image Processing, Affine Transformations,

Hierarchical Modeling, Perspective Projections

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Assigned: Saturday, April 10th
Due: Friday, April 23rd
at the beginning of class

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to discuss the problems with classmates, but please ***answer the questions on your own*.**

Name:____

Problem 1: Display Devices (10 points)

Provide a short answer to each of the following questions. In each case, you must clearly justify your answer.

(a) **(5 points)** As we'll discuss later in the quarter, three color channels are the minimum needed to create much of the range of colors that people can see. However, we can do better. For instance, combining equal amount of red and green (e.g., R=G=255, B=0) makes yellow on a display, but with a dedicated yellow phosphor or yellow LCD filter, we could make an even brighter shade of yellow (e.g., R=G=B=0, Y=255).. In general, we can increase the range and richness of the colors of a display system by adding more color channels. How would you have to modify a framebuffer and an LCD display to accommodate more color channels? Is there any disadvantage to displaying, say, 100 different channels?

(b) **(5 points)** In lecture, we discussed how double-buffering could be used to avoid drawing into the current frame while it is being displayed. Consider the idea of triple-buffering or N-buffering. Give an example of a situation where having more than two buffers would be helpful and another example where it would not make a difference?

Problem 2: Convolution filters (15 points)

In this problem, you need to design convolution filters to accomplish several different tasks. For each sub-problem, provide a filter that should accomplish the task. *Justify your choice of filter*.

(a) **(5 points)** The image you're editing is too dark, and you decide you need to amplify the value of each pixel by a factor of 2. Suggest a convolution filter that will double the value at each pixel of the image without changing it in any other way. (Technically, after scaling pixel values, they could be out of range; assume that any needed clamping will be taken care of later, after filtering).

(b) **(5 points)** While taking a photograph with your digital camera, you fail to hold the camera steady, and it translates downward while the shutter is open. You discover this later when you see that horizontal edges, in particular, have been blurred a bit (an effect called "motion blur"). You decide to filter the image so that horizontal edges are sharpened, but vertical edges are unchanged. Suggest a single convolution filter that does this.

(c) **(5 points)** After thinking a little more about the previous picture, you decide that motion blur is cool, and you want to apply it to another image. In this case, though, you want to simulate the effect of a camera translating diagonally along the x=y direction while the shutter is open. Suggest a convolution filter that would accomplish some diagonal blurring along that direction.

Problem 3: Image boundaries (20 points)

When applying a convolution filter, there are many possibilities for handling pixels near image boundaries. For each sub-problem below, discuss the consequences of the given boundary filtering approach when applying a mean filter the following simple image:

Assume that the "right" answer would correspond to the image continuing naturally beyond the boundaries so that the extended image plane would be divided into a gray half and a black half as suggested by the input image. Will a given approach below give the "right" answer all along the boundary? If not where does it give the "wrong" answer and in what way?

(a) **(5 points)** Calculate only the values of the pixels in the resulting image for which the support of the mean filter is entirely contained within the original image.

(b) **(5 points)** Pad the edges of the original image with zeros before filtering.

Problem 3: Image boundaries (cont'd)

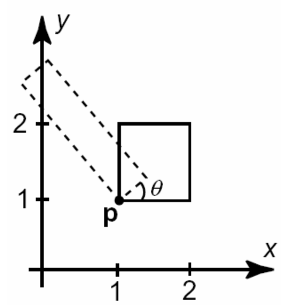
(c) **(5 points)** Reflect the original image across each image boundary before filtering. You can think of this as reflecting the image across the top, bottom, left, and right boundaries, and then, to fill the diagonal regions near the corners, you would reflect the top reflection to the left and right, and the bottom reflection to the left and right.

(d) **(5 points)** Perform a "toroidal wrap" before filtering. This kind of wrapping maps the original image onto a torus so that the left edge meets the right edge and the top edge meets the bottom edge. It is equivalent to tiling the plane with the original image.

Problem 4: Affine Transformations (17 points)

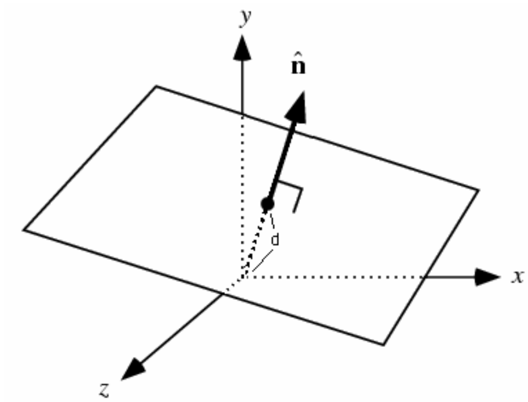
In this problem, you will solve for products of matrices to achieve a desired affine transformation in two and three dimensions. In each case, your answers should be built from individual matrices described in lecture, including rotations, scales, reflections, shears, and translations. You should specify the contents of each matrix in your final answer; however, you do **not** need to multiply all the matrices together.

(a) **(6 points)** An affine transformation has been applied to a unit square, shown below with solid lines, to transform it into the figure with dotted lines. Assume that after transformation, the height is doubled and the width is halved. Write out a product of 3x3 matrices that perform this transformation. Justify your answer.



Problem 4: Affine Transformations (cont'd)

(b) (11 points) The equation $\hat{\mathbf{n}} \bullet \bar{\mathbf{x}} = d$ describes the plane pictured below which has unit length normal $\hat{\mathbf{n}}$ pointing away from the origin and is a distance *d* from the origin (in the direction of the normal vector).

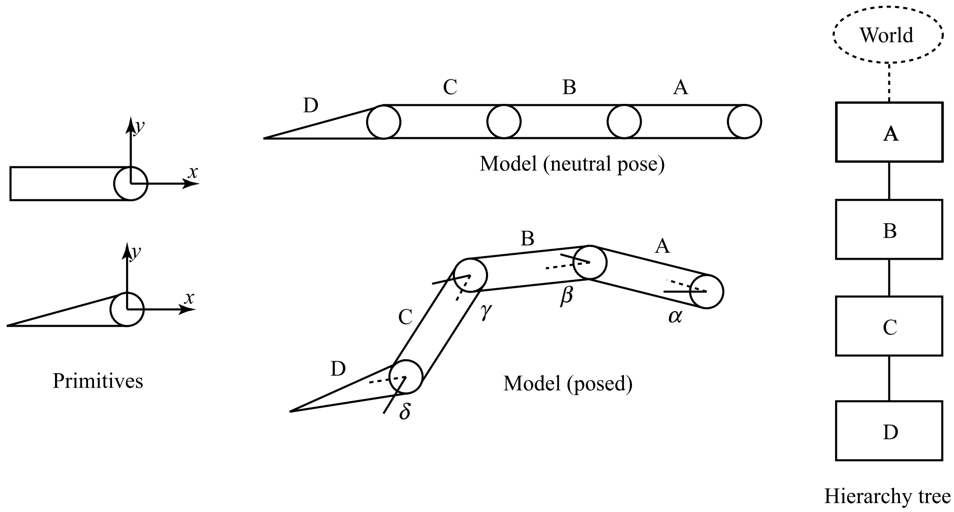


$$M_{xz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now consider a plane with normal lying in the *x*,*y*-plane. The normal will have the form (cosθ, sinθ, 0) for some θ. The equation for the plane is then $x\cos\theta + y\sin\theta = d$. Write out the product of 4x4 matrices that would perform a reflection across this plane. One of these matrices will be a reflection matrix; you must use the matrix above, which performs a reflection across the *x*,*z*-plane. Justify your answer.

Problem 5: Hierarchical modeling (17 points)

In the first part of this problem you will be using the model provided below and the corresponding hierarchy tree.



(a) **(5 points)** Write down the full transformation that **primitive D** undergoes after posing the model as shown above. You should use only the transformations $R(\theta)$**,** $T(x, y)$, and $S(x, y)$ as needed. You do not need to write out the contents of each matrix for this problem; i.e., you should leave the matrices in symbolic form. Assume that the length of **primitive A** is l_A, the length of **primitive B** is l_B, etc. The angle at each joint is defined to be the angle from the solid line to the dashed line measured in the counter-clockwise direction (right-handed coordinates).

(b) **(2 points)** Suppose that we want to use the above leg model to form an ant-like creature and you already have a complex ant body model and hierarchy tree with no legs. Describe how would you combine the two models and trees.

Problem 5: Hierarchical modeling (cont'd)

In the second part of this problem you will be working with 'your model'. Your model is the model you plan to build for the Modeler project, a simplified version of that model, or a completely different model that *could* be a built for the Modeler project. Your model must have at least four articulated levels of hierarchy. That is, you need at least three joints. The transformation that maps the model into world coordinates (e.g., the translation of the root node) does not count as a joint. The model in the previous problem qualifies as having enough hierarchical levels, but you should not, of course, use that model in this problem (it's not a very interesting model, anyway!). You may work with your partner to design the model and hierarchy you use for your answer.

(c) **(10 points)** Sketch your model in two poses and draw the corresponding hierarchy tree. One pose should be the model in a neutral state and one should have changes at each of four levels of your hierarchy along exactly one descending path through the tree. Label all the parts of the model in the neutral pose and the corresponding nodes in the tree. Label the transformations only along the edges that correspond to what changes in moving from the neutral pose to the other pose you provided. You may use generic names for the transformations, as was done in the lecture; e.g., M_{ua} was the "upper arm" transformation for the humanoid figure. You do not need to have separate sketches for each primitive of your model.

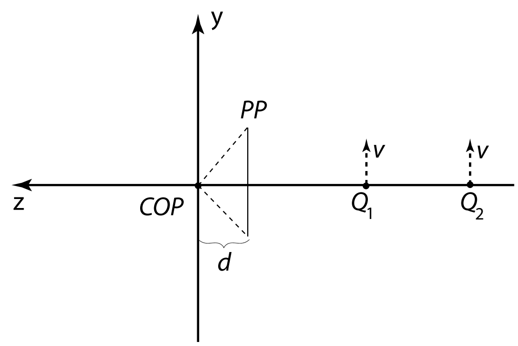
Problem 6: Projections (18 points)

The apparent motion of objects in a scene can be a strong cue for determining how far away they are. In this problem, we will consider the projected motion of points and line segments and their apparent velocities as a function of initial depths.

(a) **(7 points)** Consider the projections of two points, Q_1 and Q_2, on the projection plane *PP*, shown below. Q_1 and Q_2 are described in the equations below. They are moving parallel to the projection plane, in the positive *y*-direction with speed *v*.

$$Q_1(t) = \begin{bmatrix} 0 \\ vt \\ z_1 \\ 1 \end{bmatrix} \qquad Q_2(t) = \begin{bmatrix} 0 \\ vt \\ z_2 \\ 1 \end{bmatrix}$$

$$0 > z_1 > z_2$$



Compute the projections q_1 and q_2 of points Q_1 and Q_2, respectively. Then, compute the velocities, dq_1/dt and dq_2/dt, of each projected point in the image plane. Which appears to move faster? Show your work.

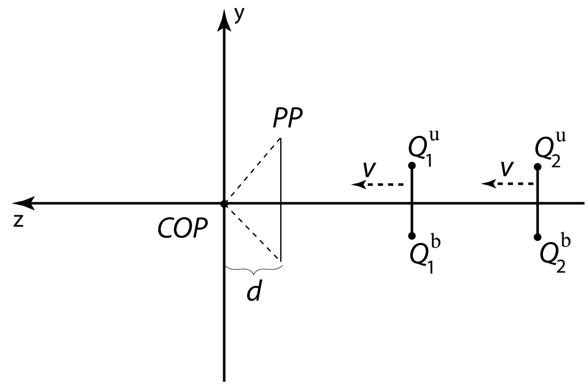
Problem 6: Projections (cont'd)

(b) **(11 points)** Consider the projections of two vertical line segments, S_1 and S_2, on the projection plane *PP*, shown below. S_1 has endpoints, Q_1^u and Q_1^b. S_2 has endpoints, Q_2^u and Q_2^b. The line segments are moving perpendicular to the projection plane in the positive *z*-direction with speed *v*.

$$Q_1^u(t) = \begin{bmatrix} 0 \\ 1 \\ z_1 + vt \\ 1 \end{bmatrix} \qquad Q_2^u(t) = \begin{bmatrix} 0 \\ 1 \\ z_2 + vt \\ 1 \end{bmatrix}$$

$$Q_1^b(t) = \begin{bmatrix} 0 \\ -1 \\ z_1 + vt \\ 1 \end{bmatrix} \qquad Q_2^b(t) = \begin{bmatrix} 0 \\ -1 \\ z_2 + vt \\ 1 \end{bmatrix}$$

$$0 > z_1 > z_2$$



Compute the projected lengths, l_1 and l_2, of the line segments. Then, compute the rates of change, dl_1/dt and dl_2/dt, of these projected lengths. Are they growing or shrinking? Which projected line segment is changing length faster? Show your work.