Homework #1

Graphics Systems, Filtering,

Transformations, Hierarchies, Projections

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Assigned: Monday, April 11th
Due: Friday, April 22nd

Directions: Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to discuss the problems with classmates, but please ***answer the questions on your own*****.**

Name:____

Problem 1: Short Answer (8 points)

(a) (2 points) A handheld computer has a display capable of true color (24 bit per pixel) output, but in order to save display memory, it uses a color index with 16 bit addresses. A more expensive model is otherwise identical, but has a larger true color frame buffer. Given that both devices operate at a display resolution of 320x200, will the more expensive model offer any advantage?

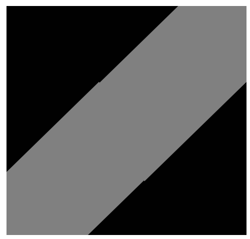
(b) (2 points) Color lookup tables are useful even with used with a true color frame buffer. Give an example of what the lookup table might be used for.

(c) (2 points) Do parallel lines remain parallel under all affine transformations?

(d) (2 points) In the graphics pipeline, why might it be preferable to use a matrix multiplication to represent translation rather than vector addition?

Problem 2: Image Boundaries (8 points)

When applying a convolution filter, there are many possibilities for handling pixels near image boundaries. For each sub-problem below, discuss the consequences of the given boundary filtering approach when applying a mean filter the following simple image:



Assume that the "right" answer would correspond to the image continuing naturally beyond the boundaries so that the extended image plane would be a gray stripe in a black plane as suggested by the input image. Will a given approach below give the "right" answer all along the boundary? If not where does it give the "wrong" answer and in what way?

(a) (2 points) Calculate only the values of the pixels in the resulting image for which the support of the mean filter is entirely contained within the original image.

(b) (2 points) Pad the edges of the original image with zeros before filtering.

Problem 2: Image Boundaries (continued)

(c) **(2 points)** Reflect the original image across each image boundary before filtering. You can think of this as reflecting the image across the top, bottom, left, and right boundaries, and then, to fill the diagonal regions near the corners, you would reflect the top reflection to the left and right, and the bottom reflection to the left and right.

(d) **(2 points)** Perform a "toroidal wrap" before filtering. This kind of wrapping maps the original image onto a torus so that the left edge meets the right edge and the top edge meets the bottom edge. It is equivalent to tiling the plane with the original image.

Problem 3: Perspective Projections (7 points)

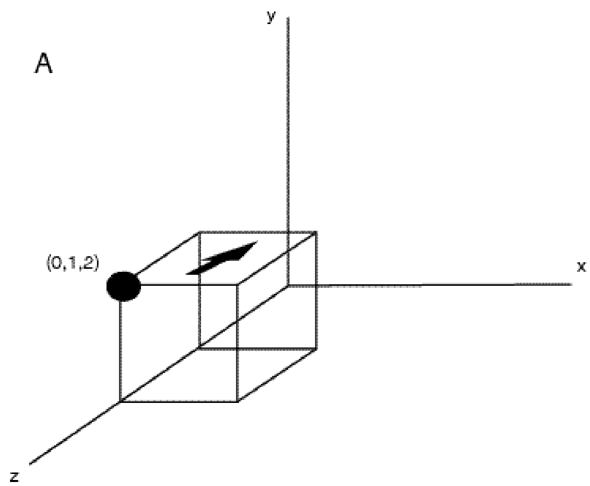
Recall the matrix for perspective projection $P(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix}$

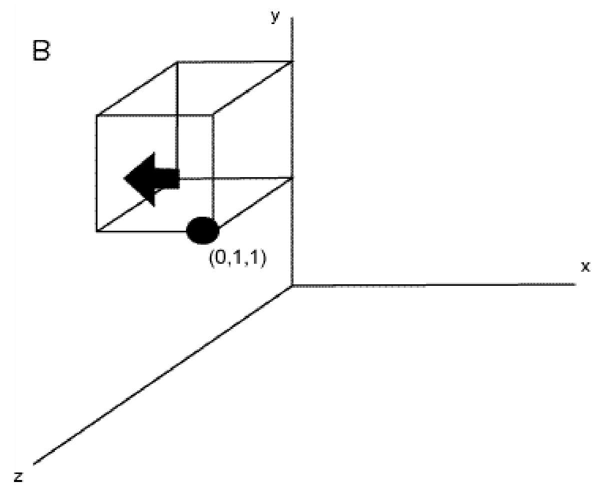
1. (5 points) The perspective projection matrix takes a set of points p_i in 3-space and maps them to points on the viewing plane $p_i' = P(d)\,p_i$. If we first apply a uniform scale by a factor s to all points in 3-space, before performing the projection indicated by this matrix $p_i' = P(d)S(s)\,p_i$, how will the projected points be different from the case where no scale is applied? Show your reasoning mathematically and explain the result geometrically (in words and/or with a sketch). ***Hint: it may be useful to think about what this looks like in two dimensions instead of three.***

2. (2 points) Using the perspective projection matrix above, we observed in class that any set of parallel lines will converge and intersect at a vanishing point. In fact, some parallel lines will ***not*** intersect, i.e., they will remain parallel, even after undergoing this perspective projection. Which lines are these? Geometrically speaking, what do all sets of these lines have in common?

Problem 4: 3D Affine Transformations (6 points)

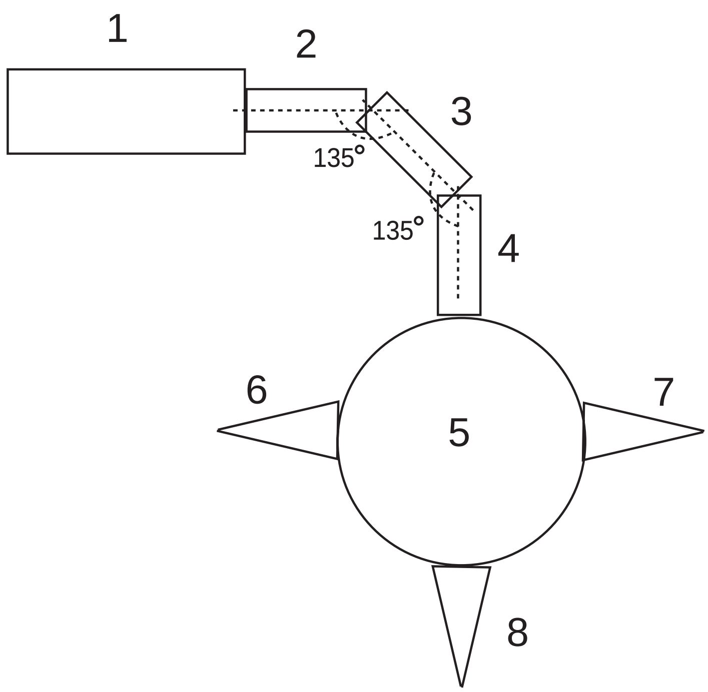
The cube shown has edges of length 1 and a diamond on one face. Determine a product of 4x4 matrices that perform the transformation from **A** to **B** as shown in the graphs below. Do not multiply the matrices out.

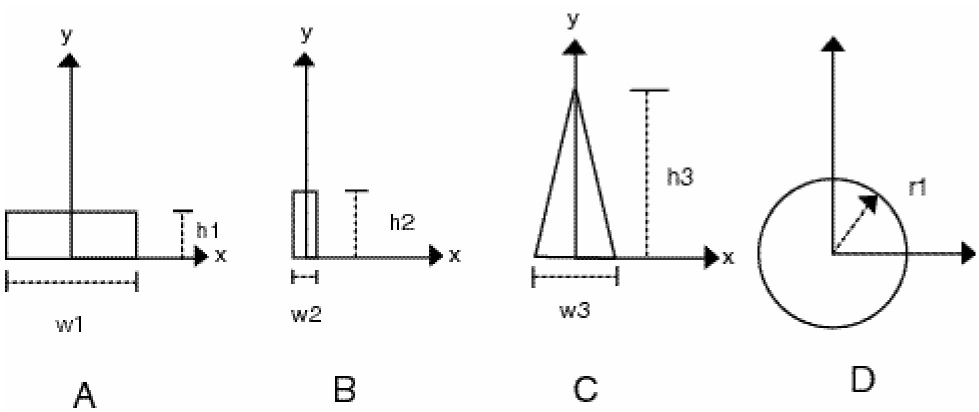




Problem 5: Hierarchical Modeling (10 points)

Suppose you want to model the mace shown below. The mace is made out of eight parts, and each part is drawn as one of the four primitives shown below.





The following transformations are also available to you:

- R(t) – rotate by t degrees (counter clockwise)
- T(a, b) – translate by (a, b)

1. (8 points) Construct a tree to specify the mace that is rooted at **1**. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that order is important!

2. (2 points) Write out the full transformation expression for the part labeled **7**.

Extra Credit: Rotations as Shear Transformations (8 points)

You are ***not*** required to do this problem! But extra points are good things, so we encourage you to give it a try.

In 2D, a rotation transformation by angle θ can be specified as a series of shear transformation matrices. Give these matrices, or if it can't be done, prove it.