Computer Science and Engineering 457

Introduction to Computer Graphics

Homework 3

Spring Quarter, 2012; University of Washington

Due Wednesday, May 23 at the beginning of class

Instructions: Do this assignment individually (NOT in partnerships). Legibly write your answers on sheets of paper. When giving explanations, make sure they are clear. If there are corrections or clarifications to these problems, they will be posted on the course website, linked from the homepage. This is version 1.01.

1. (10 points) Barycentric Coordinates.

   Consider the triangle $T = \triangle ABC$, where $A = [0, 0, 5]$, $B = [3, 0, 5]$, and $C = [0, 4, 5]$.

   Suppose this triangle is part of a mesh for which we are doing Phong shading (interpolating vertex normals). Suppose the normals have been determined to be:

   $N_A = [0, 0, 1]$, $N_B = [1/\sqrt{2}, 0, 1/\sqrt{2}]$, and $N_C = [0, 1/\sqrt{2}, 1/\sqrt{2}]$.

   Now consider a point $P = [1, 2, 5]$.

   (a) (6 points) Find the barycentric coordinates of P.

   (b) (4 points) Compute the (normalized) normal at P.

2. (20 points) Shading Polyhedra

   In Figure 2 we see a regular octahedron having six vertices A, B, C, D, E, F, and follows:

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A: [1, 0, 0]
B: [0, 1, 0]
C: [0, 0, 1]
D: [0, -1, 0]
E: [-1, 0, 0]
F: [0, 0, -1]
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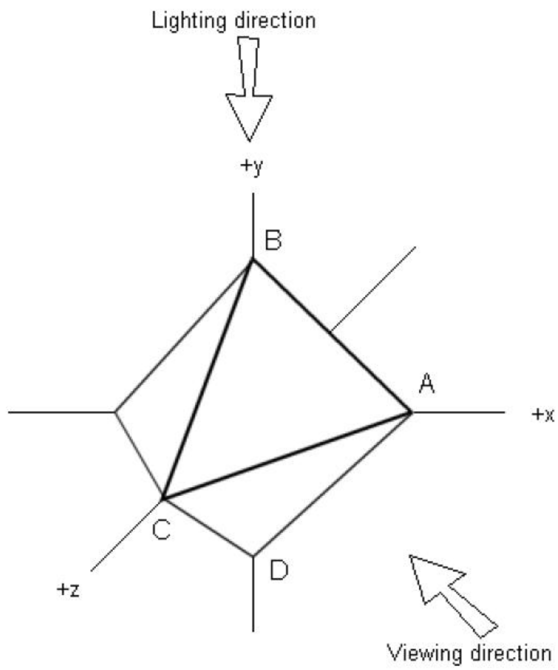
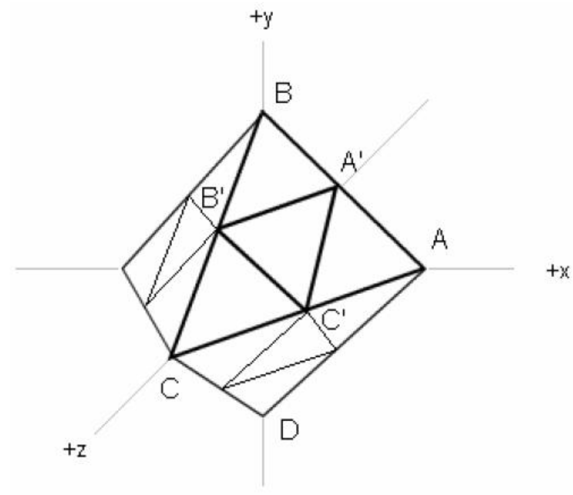


   Assume the viewpoint is infinitely far away, looking along the direction given by the vector $(-1, 0, -1)$. Assume that the scene is lit by a directional light shining in the direction $-y$, with monochromatic light of intensity 1. We'll consider the faces of the octahedron to be made of a material that has both diffuse and specular components, but no ambient or emissive components. Thus we can use the following simplified from of the Phong shading equation.

$$I = k_d L(\vec{N} \cdot \vec{L})_+ k_s L(\vec{V} \cdot \vec{R})_+^{n_s}$$

   Because we are assuming monochromaticity, we'll take k_d and k_s to be scalars. Assume they are each equal to 0.5. Assume the value of n_s is 50.



(a) (3 points) Assume now that we will render each face of the octahedron with flat shading. We will use the normals of each face, and not worry about the fact that the normals at the edges and vertices are multiply defined. Compute the unit normals for the triangle ABC and for the triangle ACD.

(b) (3 points) Now assume the octahedron is serving as a crude approximation to a sphere and that we should use smoothly varying shading on it. What should be the unit normals at A, B, and C? Show your work.

(c) (3 points) Using the normals from (b), compute the rendered intensities at those same three vertices.

(d) (2 points) If we were to use Gouraud shading with these values, what would the triangle ABC look like? (Describe this in words).

(e) (2 points) Suppose now that we use Phong-interpolated shading instead of Gouraud shading. How will the appearance of triangle ABC change?

(f) (5 points) If the octahedron is being used to approximate a sphere, we can improve the approximation by subdividing each of its triangles into four new equilateral triangles and then specifying three new vertices. This is sometimes called four-to-one triangular subdivision. For the triangle ABC, what would be the best choices for the coordinates of the new vertices A', B', and C'? See the figure. What should be the new unit normal at A'?

(g) (2 points) If the subdivision process were continued, getting closer and closer to the sphere, would the Gouraud and Phong-interpolated renderings converge to the same picture of a sphere?

3. (16 points) Parametric Curves

In the following table, put a check mark in a box if the type of curve given at the left of the box's row has the property given at the top of the box's column.

	C^2 continuous	locally controlled	interpolates all control points	bounded by convex hull of control points
Bezier				
B-spline				
C^2 interpolating				
Catmull-Rom				

Figure 1: Control points for a Bezier curve.

4. (18 points) B-Splines

   In this problem, we will explore the construction of parametric. Please write on the pages for this problem and include them with your homework solution.

   (a) (6 points) Given the Bezier control points shown in Fig. 1, construct all of the de Casteljau lines and points needed to evaluate the curve at $u = 1/3$. Mark this point on your diagram and then sketch the path the Bezier curve will take. The curve does not need to be exact, but it should conform to some of the geometric properties of Bezier curves (convex hull condition, tangency at endpoints).

   (b) (6 points) Given the de Boor points shown in Fig. 2, construct all of the lines and points needed to generate the Bezier control points for the B-spline. Assume that the first and last points are each repeated three times, so that the spline is endpoint interpolating. You must mark each Bezier point (including any that coincide with a de Boor point) with an X, but you do not need to label it (i.e., no need to give each Bezier point a name). Sketch the resulting spline curve, respecting the properties of Bezier curves noted above.

   (c) (6 points) Given the Catmull-Rom control points shown in Fig. 3, construct all of the lines and points needed to generate the Bezier control points for the Catmull-Rom curve. Use a tension value of $\tau = 1/2$. As-

Figure 2: Control points for a B-spline curve.

sume that the first and last points are each repeated two times. You must mark each Bezier point (including any that coincide with a Catmull-Rom control point) with an X, but you do not need to label it (i.e., no need to give each Bezier point a name). Sketch the resulting spline curve, respecting the properties of Bezier curves noted above.

Figure 3: Control points for a Catmull-Rom spline curve.