Lecture 4:
Image Processing

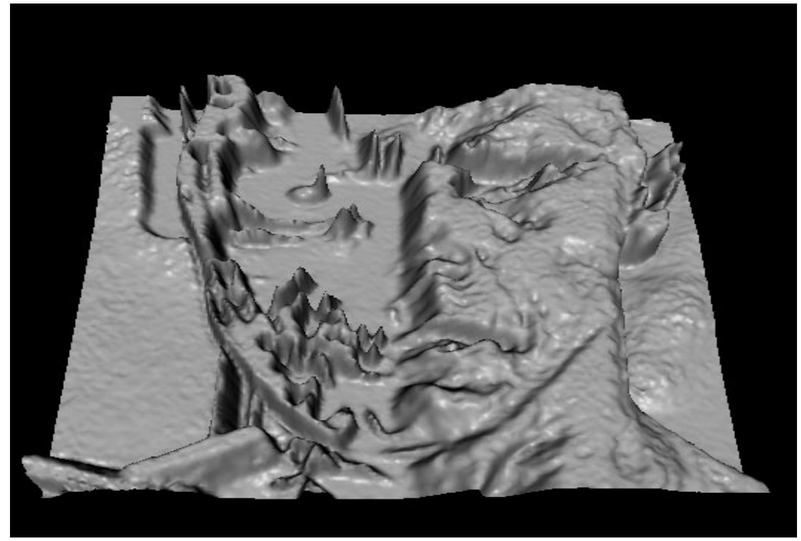
Definitions

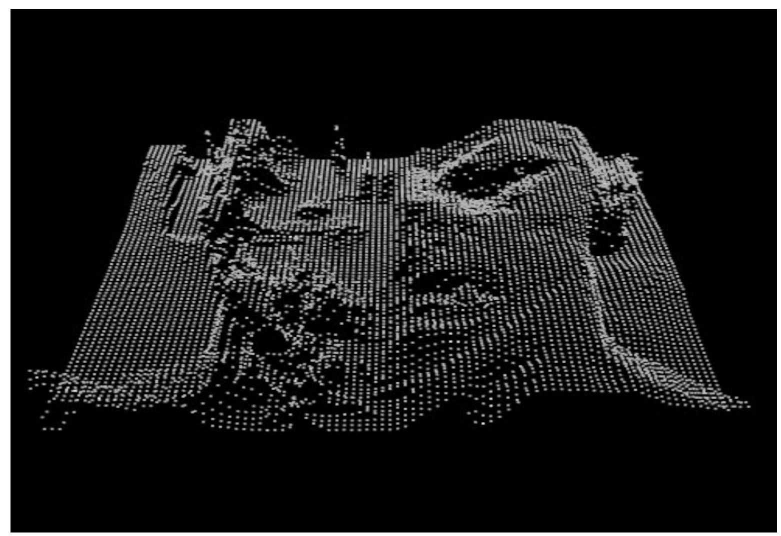
- Many graphics techniques that operate only on images
- **Image processing**: operations that take images as input, produce images as output
- In its most general form, an **image** is a function *f* from R^2 to R
  - $f(x, y)$ gives the intensity of a channel at position (x, y)
  - defined over a rectangle, with a finite range: $f: [a,b] \times [c,d] \rightarrow [0,1]$
  - A color image is just three functions pasted together:
    - $\boldsymbol{f}(x, y) = (f_r(x, y), f_g(x, y), f_b(x, y))$

Images

- In computer graphics, we usually operate on **digital** (**discrete**) images
  - **Quantize** space into units (pixels)
  - Image is constant over each unit
  - A kind of step function
  - f: $\{0 \ldots m\text{-}1\} \times \{0 \ldots n\text{-}1\} \rightarrow [0,1]$
- An image processing operation typically defines a new image *f'* in terms of an existing image *f*

Images as Functions



Pixel-to-pixel Operations

- The simplest operations are those that transform each pixel in isolation
  $f'(x, y) = g(f(x,y))$
- Example: threshold, RGB $\rightarrow$ greyscale

Pixel Movement

- Some operations preserve intensities, but move pixels around in the image

  $f'(x, y) = f(g(x,y), h(x,y))$

- Examples: many amusing warps of images

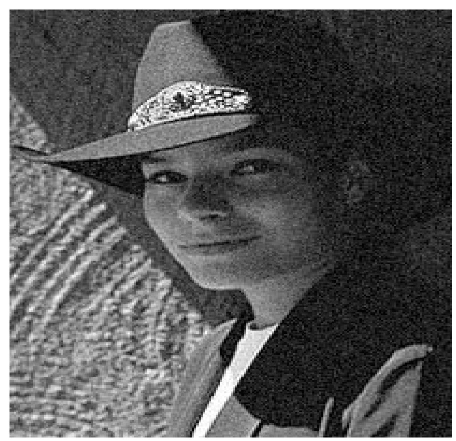
Noise

Common types of noise:

Original

Salt and pepper noise

Impulse noise



Gaussian noise

Noise Reduction

- How can we “smooth” away noise?

Convolution

- Convolution is a fancy way to combine two functions.
  - Think of *f* as an image and *g* as a "smear" operator
  - *g* determines a new intensity at each point in terms of intensities of a neighborhood of that point

$$
\begin{aligned}
h(x,y) &= f(x,y) * g(x,y) \\
&= \int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy'
\end{aligned}
$$

- The computation at each point (*x*,*y*) is like the computation of cone responses

Discrete Convolution

- Since digital images are like step functions, integration becomes summation. We can express convolution as a two-dimensional sum:

$$\begin{aligned} h[i,j] &= f[i,j] * g[i,j] \\ &= \sum_k \sum_l f[k,l]g[i-k,j-l] \end{aligned}$$

Convolution Representation

- Since *f* and *g* are defined over finite regions, we can write them out in two-dimensional arrays:

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

X .2	X 0	X .2
X 0	X .2	X 0
X .2	X 0	X .2

Mean Filters

- How can we represent our noise-reducing averaging filter as a convolution diagram?

Using Mean Filters

Gaussian noise | Salt and pepper noise

3x3

5x5

7x7

Gaussian Filters

- Gaussian filters weigh pixels based on their distance to the location of convolution.

$$g[i,j] = e^{-(i^2+j^2)/(2\sigma^2)}$$

- Blurring noise while preserving features of the image
- Smoothing the same in all directions
- More significance to neighboring pixels
- Width parameterized by σ
- Gaussian functions are separable

Using Gaussian Filters

Gaussian noise | Salt and pepper noise

3x3

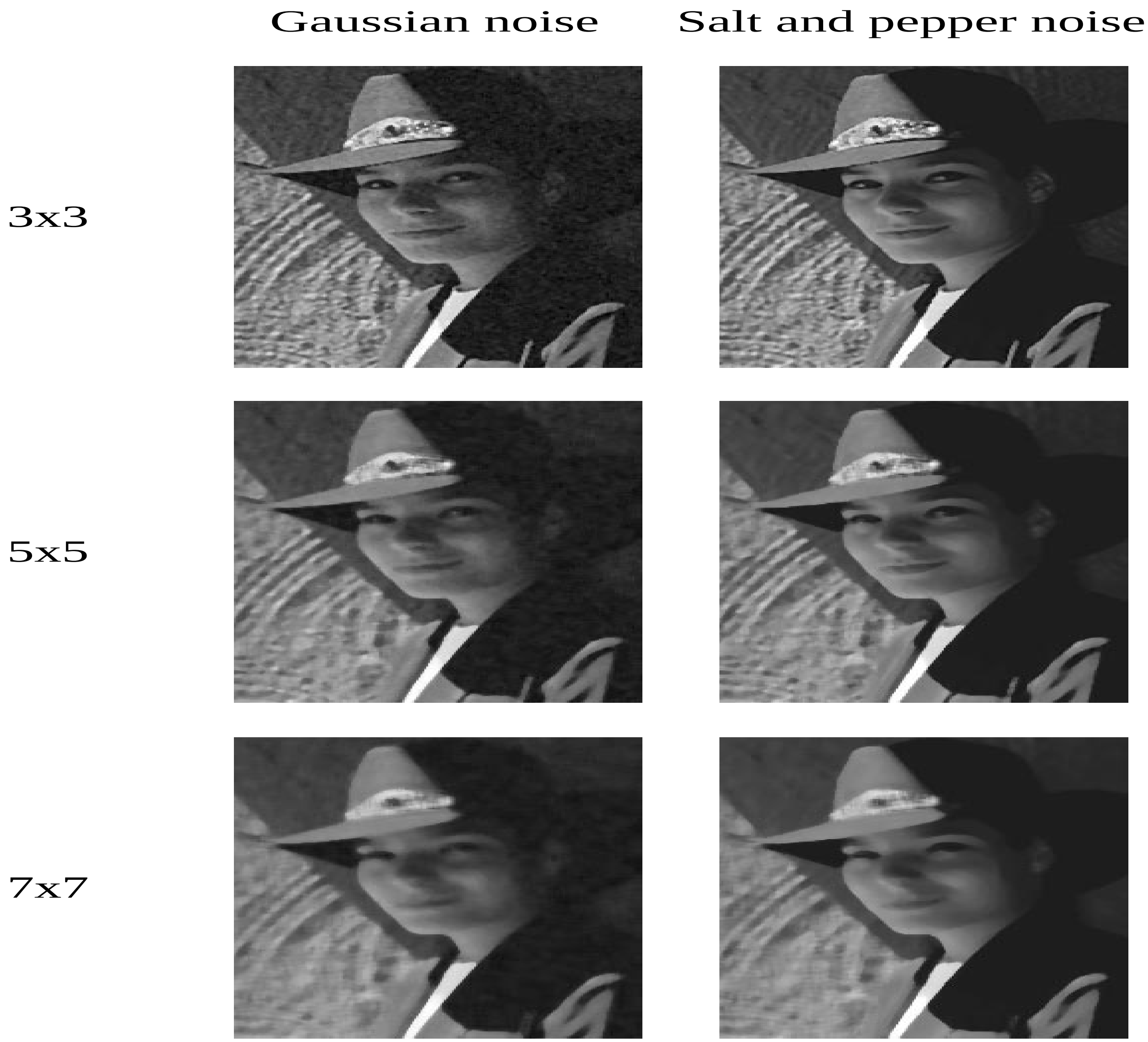
5x5

7x7

Median Filters

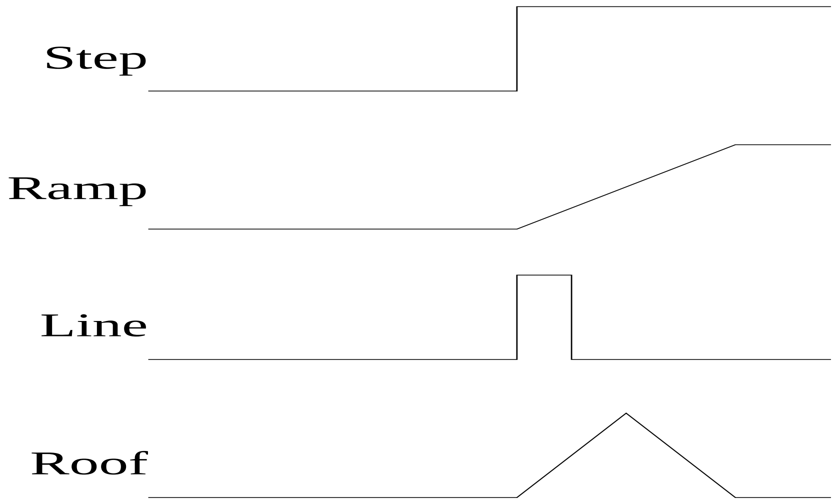
- A **Median Filter** operates over a *k*x*k* region by selecting the median intensity in the region.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Using Median Filters



Edge Detection

- One of the most important uses of image processing is **edge detection**
  - Really easy for humans
  - Really difficult for computers
  - Fundamental in computer vision
  - Important in many graphics applications
- What defines an edge?



Gradient

- The **gradient** is the 2D equivalent of the derivative:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

- Properties of the gradient
  - It's a vector
  - Points in the direction of maximum increase of *f*
  - Magnitude is rate of increase
- How can we approximate the gradient in a discrete image?

Edge Detection Algorithms

- Edge detection algorithms typically proceed in three or four steps:
  - Filtering: cut down on noise
  - Enhancement: amplify the difference between edges and non-edges
  - Detection: use a threshold operation
  - Localization (optional): estimate geometry of edges beyond pixels

Edge Enhancement

- A popular gradient magnitude computation is the **Sobel operator**:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- We can then compute the magnitude of the vector (s_x, s_y)

Using Sobel Operators


Original


Smoothed


Sx + 128


Sy + 128


Magnitude


Threshold = 64


Threshold = 128

Second Derivative Operators

- The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.
- An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.
- We can find these by looking for zeroes in the *second* derivative
- Using similar reasoning as above, we can derive a Laplacian filter, which approximates the second derivative:

$$\triangle^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Zero values in the convoluted image correspond to extreme gradients, i.e. edges.

Summary

- Formal definitions of image and image processing
- Kinds of image processing: pixel-to-pixel, pixel movement, convolution, others
- Types of noise and strategies for noise reduction
- Definition of convolution and how discrete convolution works
- The effects of mean, median and Gaussian filtering
- How edge detection is done
- Gradients and discrete approximations