CSE/EE 461
Assignment 3
Due Friday October 24, 1997

Your solutions must contain the derivation of your answers.

1. This problem explores the properties of CRC.

   Consider the CRC generator $x^5 + x^4 + x^2 + 1$.

   (a) What is the binary representation of this generator?

   (b) Why does the CRC based on this generator detects all bursts of length ≤ 5.

   (c) Why does the CRC based on this generator detects all k-bit errors where k is odd.

   (d) If the message to be sent is 1001110110 then what bit string is actually transmitted. Show the division that is used to calculate the transmitted message.

   (e) Give an example of a message which is received incorrectly because a 2 bit error goes undetected. Hint: start with an error in the first bit and try to find where the second error must be for the generator to divide the received data evenly.

2. This problem explores the problem of queuing delay and packet loss at a multiplexer, modeled by simple queue.

   Suppose we have a multiplexer which has Poisson arrivals at a rate λ and Poisson service rate of μ. What this means is that for small δ the probability of an arrival of a packet during a time interval δ is approximately $\lambda\delta$ and the probability of no arrivals is approximately $1 - \lambda\delta$. The probability of multiple arrivals is negligible for small δ. The multiplexer has a bounded queue of size N and packets are serviced (transmitted) in such a way that if the queue is not empty then with probability approximately $\mu\delta$ a packet is removed from the queue in a time interval of length δ and with probability $1 - \mu\delta$ no packet is not removed in a time interval of length δ. With negligible probability does a packet arrive and another packet depart from the queue in a time interval of length δ. If the queue is full and a packet arrives then it is lost. This multiplexer can be modeled by a finite state Markov chain with states $s_0, s_1, s_2, ..., s_N$ where state s_i represents the multiplexer having exactly i packets in its queue. For small δ and $i < N$, the probability of going from state s_i to s_{i+1} is approximately $\lambda\delta$ and the probability of going from state s_{i+1} to s_i is approximately $\mu\delta$. For small δ and $i > 0$, the probability of staying in state s_i is $1-(\lambda+\mu)\delta$ and the probability of staying in state s_0 is $1-\lambda\delta$. All other transition probabilities are negligible for small δ.

(a) Let π_i be the long term probability that the queue is of size i. Assuming δ goes to zero, compute π_i for $i \geq 0$ as a function of λ, μ, N, and i. This involves solving a system of linear equations. There will be different answers depending on whether or not $\lambda < \mu$.

(b) Give a nice expression in λ, μ, and N for the expected queue length where the expected queue length is $\sum_{i=0}^{N} i\pi_i$.

(c) Give a nice expression in λ, μ, and N for the percentage of packets that are lost due to queue overflow.

(d) For $\lambda < \mu$ the expected utilization U of the outgoing channel from the multiplexer can be expressed as $U = \lambda/\mu$ and N, that is, the arrival rate over the departure rate (when the queue is not empty). Plot the expected queue length for values of $U = .1, .2, .3, ..., .9$ and $N = 10$.

(e) Plot the percentage expected packet lost as a function of U for values of $U = .1, .2, .3, ..., .9$ and $N = 10$.

Some useful formulas are, for $|\alpha| < 1$,

$$\sum_{i=0}^{N} \alpha^i = \frac{1-\alpha^{N+1}}{1-\alpha},$$

$$\sum_{i=0}^{N} i\alpha^i = \frac{\alpha-(N+1)\alpha^{N+1}+N\alpha^{N+2}}{(1-\alpha)^2}.$$