CSE/EE 461
Assignment 5
Due Friday, November 14, 1997

Your solutions must contain the derivation of your answers.

1. This problem uses some elementary probability and algebra. The goal of the problem is analyze and understand a simple mechanism for improving the utilization of a broadcast channel by allowing transmitters to use more time when they finally are able to grab the channel. The analysis in this problem can be challenging. Be sure to start it early and ask questions for help.

   Suppose we have a broadcast channel with n stations. The channel is divided into time slots and all stations can listen to the channel to hear what is going on. When a station attempts to transmit there are two possibilities: (i) a collision, in which case the transmitter immediately halts it transmission or (ii) a successful transmission, in which case the transmitter transmits for k time slots. While a station is successfully transmitting all other stations do not try. Otherwise, each station tries to transmit with probability p. From our previous assignment, homework 2, problem 2, we computed, when stations are free to transmit, the probability (i) that no stations transmit, q_0, (ii) exactly one station transmits, q_1, and (iii) more than one station transmits and a collision occurs, q_*.

   (a) Conceptually, the current slot can be in k possible states called $s_0, s_1, ..., s_{k-1}$. In state s_0 all stations are free to transmit in the slot each with probability p. In state s_i, for $i > 0$, some station is in the process of transmitting in its $(i+1)$-st slot. Let π_i be the probability that a slot is in state s_i. To begin with we must have $\pi_0 + \pi_1 + ... + \pi_{k-1} = 1$, that is, all the probabilities must add up to 1. It must also be the case that $\pi_1 = \pi_2 = ... = \pi_{k-1}$, since if a given slot is in state s_i, for $i \geq 1$, then the previous slot must have been in state s_{i-1} and there is no other way for the slot to be in state s_i. Note, also that there are basically two ways for a slot to be in state s_0. Either the previous slot was in state s_{k-1} or the previous slot was in state s_0 and no successful transmission occurred. Write one equation in π_0 and π_{k-1} which relates these two quantities. Now, use all the equations above to solve for $\pi_0, \pi_1, ..., \pi_{k-1}$ as a functions of k, p, and n.

   (b) Clearly, bandwidth is wasted whenever no station tries to transmit or when there is a collision. On the other hand, bandwidth is utilized when exactly one station is transmitting. Write an expression for the expected utilization of the broadcast channel as a function of k, p and n? (Hint: the expected utilization can be written in terms of the probabilities $\pi_0, \pi_1, ..., \pi_{k-1}$ which were calculated above.) For $n = 10$ and $p = 1/10$, plot the expected utilization for $k = 1, 2, 3, ..10$.