Channel Capacity

A transmission channel represents a financial investment - the goal of an engineer is to derive as much benefit as possible from it. For the case of digital transmission, this is done by maximizing the information transfer across the system.

How much information in bits can be transmitted (in simplified terms!) depends on:

- •bandwidth
- •signal-to-noise ratio
- •number of signal levels.

Shannon's Theorem and Communication System Efficiency

Shannon shows that for the additive white Gaussian noise channel, coding schemes exist for which it is possible to communicate data ***error-free*** over the channel provided the system ***data rate in bits per second*** is less than or equal to what he called the ***channel capacity C***. If the data rate is greater than that channel capacity, then error-free transmission is not possible.

The channel capacity C, in bits per second, is defined by

$$\boldsymbol{C} = \boldsymbol{W}\log_2(1+\boldsymbol{S}/\boldsymbol{N})$$

Where ***W*** is the fixed bandwidth of a system, ***S*** is the received signal data power in Watts, and ***N*** is the system noise power.

Spectral Occupancy and Bandwidth Considerations

Shannon's theorem clearly suggests two factors that are important in evaluating the efficiency of any communication system:

- •the bandwidth required to transmit the chosen signalling or modulation technique
- •the energy per bit-noise spectral density ratio.

An Interesting note on Bandwidth

Bandwidth definition (we will use the simple difference between the highest and the lowest channel frequency, however) …. Bandwidth ...

Can in fact be given in many different ways (see for instance Scholtz: How do you define bandwidth, Proc. Of the International Telemetering Conference, 1972.)

E.G.: Bandwidth is ...
The width, in Hz, of an ideal rectangular filter that is necessary to pass a pre-specified percentage of the total signal energy. If one is referring to baseband signaling - that is, prior to any modulation of a carrier, then the ideal filter is a low-pass filter. If one desires to specify the bandwidth requirements at the carrier level, then this ideal filter becomes a band-pass filter.

Notes to Shannon's Theorem

When using Shannon's formula, remember that bits-per-second and baud are interchangeable in binary systems.

Also, Shannon's original formulation of the theorem (Shannon, A Mathematical Theory of Communication,Bell. Syst. Tech. J. 1948) was somewhat different then our simplified version.

Nyquist's approach

The big question in advanced design is how to increase the data rate and keep the error rate reasonable.
One important item that Shannon's formula did not take into consideration is intersymbol interference. A major problem of a pulse in a band-limited channel is that the pulse tends not to die out immediatelly, and a subsequent pulse is interfered with by "tails" from the proceeding pulse.

Nyquist provided another approach to the data rate problem, this time using intersymbol interference as a limit. This resulted in the definition of the so-called Nyquist rate, which is 2***W*** elements per second:

Data Rate (in elements per second) = 2***W***,

where ***W*** is the bandwidth in Hz of a band-limited channel.

Nyquist's approach: Discussion

(adapted from Freeman, Telecommunication Transmission Book)

In binary transmission we are limited to $2W$ bits per second (as one symbol is one bit).
The key here is that if we have restricted ourselves to binary transmission (1 bit per symbol), then we are limited to $2W$ bps no matter how much we increase the signal-to-noise ratio.
The Shannon's formula however indicates that we should be able to increase the information rate by increasing signal to noise ratio.
The way to attain the higher value of C is to replace the binary transmission system with a multilevel system. Such a channel can pass (given M signal levels)

Data Rate (in bits per second) = $2W \log_2 M$ bps.

Nyquist's approach: Final Remarks

(adapted from Freeman, Telecommunication Transmission Book)

One last remark on these two fundamental equations.

Increasing data rate by increasing the M in Nyquist's formula is done at the expense of the signal-to-noise ratio. As M increases (as the number of levels of signal increases), so must the signal-to-noise ratio increase to maintain a fixed error rate!

For some fine **examples** on these issues pls. see William Stallings, Computer Networks book.